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ABSTRACT

This programed instruction study guide is one of a series that form a first-year algebra course. Structured in a multiple-choice question-answer format with scrambled pages, it is intended to be used in conjunction with a computer-managed instructional system. Volume 1 includes general instructions for working with this system, and then covers the following topics in algebra: number line, comparing numbers, sets and set membership, and subsets. Reading and homework assignments are taken from the text "Modern Algebra - Book I" by Dolciani. (Related documents are SE 015 854 - SE 015 870.) (DT)

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PROGRAMMED MATH CONTINUUM

level one

ALGEBRA

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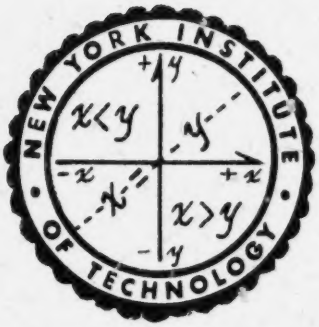

VOLUME

1

NEW YORK INSTITUTE OF TECHNOLOGY
OLD WESTBURY, NEW YORK

FILMED FROM BEST AVAILABLE COPY

SE 015 856

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P R O G R A M M E D M A T H C O N T I N U U M

LEVEL ONE

A L G E B R A

VOLUME I

New York Institute of Technology

Old Westbury - New York

PREFACE

This volume is one of a set of 18

that form a complete course

in

ALGEBRA - LEVEL ONE

The volume has been structured

in a multiple choice question-answer format,

with the pagination scrambled

and

is to be used in conjunction with

a program control console

utilizing

punch card input.

It is one exhibit in the demonstration of a model

developed under the direction of

the U.S. Department of Health Education and Welfare

Project 8-0157

at the

New York Institute of Technology

Westbury, New York

VOLUME 1

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IN THE STUDY GUIDE:

QUESTION:	SEGMENT:	IS ON PAGE:
Introduction Material		1 - 17
1	2	$\frac{18}{1}$
1	3	$\frac{68}{1}$
1	4	$\frac{110}{1}$
1	5	$\frac{149}{1}$

VOLUME 1

This volume covers the following material

as shown in this excerpt from the Syllabus.

SEGMENT	DESCRIPTION	REFERENCE BOOK SECTION		
		DOLCIANI	DRESSLER	DODES
1	Introduction: general instructions			
2	Representing numbers on a line	1-1 (1-6)	1-3 (1-1) (1-2)	3-2 4-4 4-6
3	Comparing numbers, = or	1-2 1-3	1-4	2-5 3-1
4	Membership in a set Kinds of sets	1-4 1-5 1-6	2-1 2-2 2-6 3-2 5-10	1-2 (1-3)
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D

READING ASSIGNMENT

VOLUME 1

Before you begin to answer the questions in this STUDY GUIDE you should read the pages indicated.

SEGMENT	FROM PAGE	TO PAGE
1		
2	1 (16)	5 (17)
3	5	10
4	10	17
5	18	19

Modern Algebra Book I
Dolciani, Berman and Freilich
Houghton Mifflin, 1965

Read EVERYTHING contained in these pages.

EXAMINE every illustrative problem

Write in your NOTEBOOK:

1) Every RULE that has been stated
2) Every DEFINITION that has been presented
3) Solve at least ONE PROBLEM of each type covered in the lesson.

If you wish additional information for enrichment purposes consult:

Algebra I
Dodes and Greitzer
Hayden Book Co., 1967

You will be given additional notes at various places in the STUDY GUIDE. These, too, should be entered in your NOTEBOOK.

E

HOMEWORK ASSIGNMENT

VOLUME NO. 1

BOOK: DOLCIANI

HOMEWORK QUESTION NO.	PAGE NO.	EXAMPLE NUMBER	MBO REFERENCE
1	3 - 4	1 , 2 , 3 , 4	01215
2	4	7 , 8 , 9 , 10	01215
3	17	1 , 2 , 3 , 4	01230
4	17	5 , 6 , 7 , 8	01230
5	17	9 , 10 , 11 , 12	01230
6	6	2 , 3 , 4 , 6	01310
7	7	11 , 12 , 13 , 15	01331
8	9	1 , 2 , 6 , 7	01310 , 01320
9	9	12 , 14 , 18 , 21	01310 , 01320
10	10	25 , 26 , 36 , 38	01330
11	12	27 , 28 , 29 , 30	01410
12	14	1 , 2 , 3 , 4	01421 , 01430
13	15	6 , 7 , 8 , 9	01421 , 01430
14	15	11 , 12	01421 , 01430
15	15	14 , 15	01421 , 01430
16	19	1 , 2 , 3 , 4	01512
17	19	5 , 6 , 7 , 8	01510
18	19	9 , 10	01520
19	19	11 , 12	01520
20	19	13 , 14	01520

GENERAL INSTRUCTIONS

Ask your teacher for:

PUNCH CARD
PROGRAM CONTROL
ANSWER MATRIX

When you are ready at the PROGRAM CONTROL

Insert the PUNCH CARD in the holder
Turn to the first page of the STUDY GUIDE
Read all of the instructions
Read the First Question

Copy the question
Do your work in your notebook
Do all of the computation necessary
Read all of the answer choices given

Choose the Correct answer
(remember, once you've punched the card
it can't be changed)

Punch the card with the STYLUS

Read the instruction on the PROGRAM CONTROL
(it tells you which page to turn to)

TURN TO THAT PAGE:

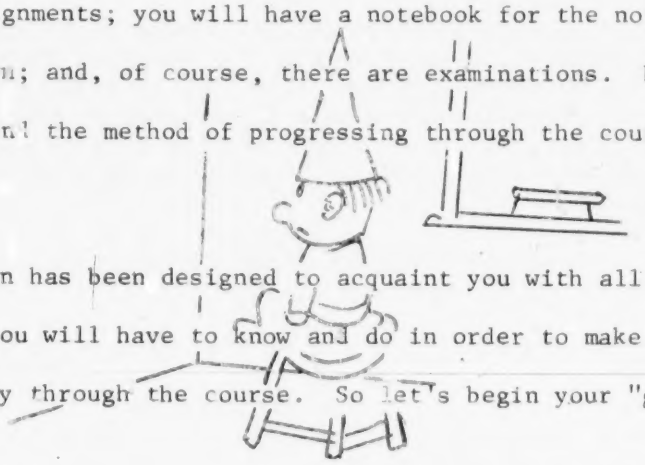
If your choice is <u>not correct</u> you will be given additional hints, and will be directed to return to the question and to choose another answer.

If your choice is <u>correct</u> then you will be directed to proceed to the next question located immediately below, on the same page.

If you have no questions to ask your teacher now, you can turn the page and begin. If you have already completed a SEGMENT turn to the beginning of the following segment;

CHECK THE PAGE NUMBER BY LOOKING AT THE TABLE OF CONTENTS

1. This is a course in ELEMENTARY ALGEBRA which is not too different from other subjects you have studied. There is a textbook; there are homework assignments; you will have a notebook for the notes you must take down; and, of course, there are examinations. However, you will find the method of progressing through the course quite different.



This first section has been designed to acquaint you with all of the things that you will have to know and do in order to make your way satisfactorily through the course. So let's begin your "guided tour":

1.1 The work of the entire course is divided into 18 parts, called VOLUMES (VOL). Each Volume consists of 5 parts, called SEGMENTS (SEG). Every VOL. deals with material which, under normal circumstances, would be covered in a two week period. However, if you are permitted to proceed at your own pace or on your own time schedule, you could probably finish a Volume in much less time.

This book, which you are now reading, is called a STUDY GUIDE, (SG). This is VOL. 1 of the SG; this portion of VOL. 1 is called SEG 1.

In addition to the SG, you will need:

(1) The basic text: Modern Algebra Book 1
By Dolciani, Berman and Freilich
Houghton Mifflin, 1965

(2) The remedial text: Comprehensive Ninth Year Mathematics
by Dressler
Amsco, 1966

(3) A Notebook: ...Where you will record specific items for further study.

And something new:

(4) Punch cards-

(5) Program control- You'll learn about these in a few minutes.

ABOUT THE PUNCH CARD:

Your progress through this course will be followed very carefully by the computer. It will record, for example, which questions you found easy and which caused you some trouble. In fact, it will keep track of all of the students in many ways and will analyze their various performances in such a way that your teacher will be able to know immediately just where extra attention should be directed to help all the students do as well as they can.

The way the computer finds out about you is through the PUNCH CARD. Those holes all mean something.

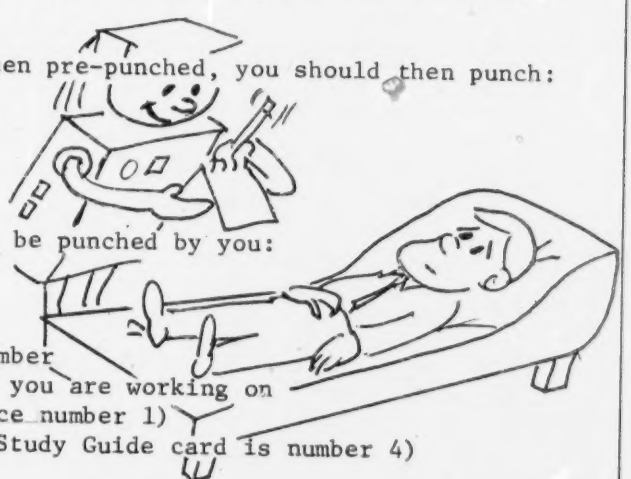
When the punch card is placed in the PROGRAM CONTROL, with the proper ANSWER MATRIX, you will see that as you punch a hole in response to a question in the SG, a light will go on in the PROGRAM CONTROL, the page to which you should turn will then be illuminated. So you see, the PUNCH CARD is very important.

HOW TO PREPARE THE PUNCH CARD:

(1) Take your PUNCH CARD and print: your LAST NAME, and FIRST NAME above the word "date."

If the card has not already been pre-punched, you should then punch:

(2) Your Course student number
(3) This Course number
(4) Your Student I.D. number



The following information must be punched by you:

(5) Month number
(6) Day number
(7) Last two digits of the Year Number
(8) The Sequence number of the SEG you are working on
(VOL. 1, SEG. 1 has Sequence number 1)
(9) The type of Input card. (The Study Guide card is number 4)
(10) The Volume Number.
(11) The Segment Number.

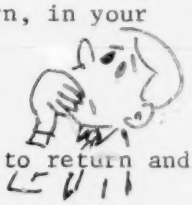
NOTE: All of this information must be entered; if not, the computer will reject the card.

Punch the necessary information, being S L O W and C A R E F U L so as to punch only those holes which you intend to punch. Remember, you cannot make corrections, once you have punched the wrong hole. Check to see that the program control device shows that it has the answer matrix for volume 1, segment 1.

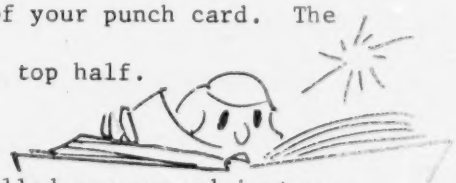
1.3 You are now ready to see how the study guide will work. Read question 1, and decide on an answer. Then examine the four choices offered.

Question 1: What is the product of 6 and 2 ?
(A) 8 (B) 12 (C) 4 (D) 3

Let us suppose that you believe the correct answer is 8, that is choice A. You would use the stylus to punch the hole on line 1 at A. Please do this now. As you see, a number appears on the program control device, the number $\frac{7}{2}$. This means turn, in your study guide, to page 7, bottom half. Do this now.



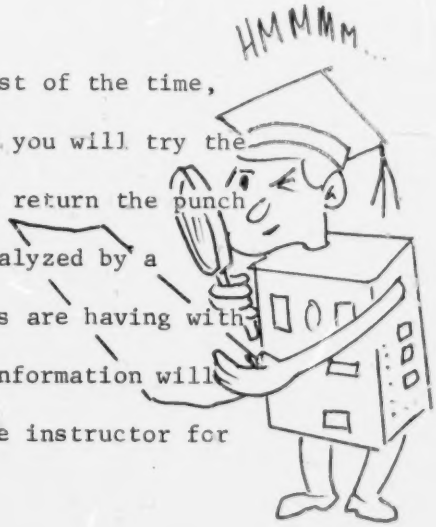
1.4 You see how the program control has directed you to return and try again, as well as telling you why your choice was not correct. Of course, looking back at question 1, you know that choice B is the correct answer. Punch B on line 1 of your punch card. The program control shows $\frac{9}{1}$. Turn to page 9, top half.



1.5 The procedure of this study guide is called programmed instruction. The purpose of the study guide is what its name indicates; it is a guide to your study. This is not a test in disguise. As you answer questions in the study guide, you should go back to your textbook, notebook, or material in the study guide itself, if you need to look up something which you are not sure of. The effect is that you will go over all the material of the course, in easy stages, with special emphasis on the more important points. The important points which the course contains are called M.B.O.'s, and each question you meet is designed to give you practice in dealing with one of these points.

1.5 Many abbreviations are used in this course. I'm sure you are curious about "M.B.O."

The letters "MBO" , stand for "Measurable Behavioral Objectives". They are the ideas, definitions and skills that are to be taught, hence they are called "objectives". Since they are stated in a way that questions can be asked about them, they are called "measurable". Because you have to demonstrate that you understand these, and can use them they are called "behavioral".



You will get the right answer on the first trial most of the time, but if you make a mistake you will find out why and you will try the problem again. At the end of the segment, you will return the punch card to your instructor. The cards will then be analyzed by a computer to find out the difficulties which students are having with the material of the course. The analysis of this information will indicate what remedial work should be planned by the instructor for the benefit of the students.

1.6 In general, at the beginning of a segment, you are directed to read a specific portion of Dolciani. In reading this, you MUST work out, on paper, any illustrative problems which the text contains, so that you are sure you can do the problem being demonstrated. Remember that the author may not have written every step as you would, so work on your own paper and try to follow the work shown in the book. In addition to the reading assignment in the text, there may be some notes in the study guide itself. You should enter in your notebook those items which seem to be of special importance, or those items which you may be directed to write down.

(continued on page 5)

Then, before you proceed with the segment, the instructor will insert the ANSWER MATRIX for your particular segment into the program control device. You will complete the necessary information items on a PUNCH CARD, insert it into the program control device, and then punch the necessary information into the card, using your stylus.

Now you proceed to the questions. You must take the questions in order. You'll have no choice, since the pages have been scrambled. That is, after question 1, you do not go ahead to the next page in order, but to whatever page the program control directs you. Each question will offer four choices, of which one, and only one, is correct. The three other choices, called distractors, are answers which you might well arrive at if your understanding of the principle were not completely correct. For every choice which you punch on your card, the program control directs you to a specific half of a particular page. Here you find a statement of why you were wrong (if you were), or a statement that you were right with instructions to proceed. This half-page is called a REM, (this is short for REMEDY) and the program control will direct you to a particular REM for any answer you choose for a question in the study guide. It should be of some help to you.

1.7 After this page, you must follow the instructions given to you by the program control. You must not go page by page, since the material is not in order.

Please remember, DO NOT GUESS:

This guide is intended to help you to learn and practice correctly.

You should go back and reread pages 1 through 5 as often as necessary. When you have considered the question, punch your answer on the punch card and the program control will tell you where to turn next.

1.8 IF YOU " LOST YOUR PLACE "

If you lose your place for any reason, insert your stylus in the last hole that you had punched on your card. This will not affect any record that's being kept. The program control will put you back on the track.

Now turn to page $\frac{7}{1}$

Question 2: What type of instructional device is this book?

(A) M.B.O. (B) punch card

(C) answer matrix (D) study guide

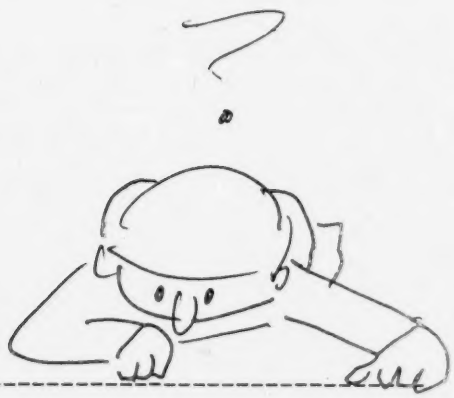
When you have decided on your answer, punch the letter of your choice on line 2 of the punch card, and follow the directions of the program control.

You have found the sum, but you were asked for the product, not the sum. The product of two numbers is the result obtained by multiplication.

Return to page 3, section 1.4

8/1

If you will refer to page 1, section 1.1, you will find that the study guide contains 18 volumes. You are correct.



Now proceed to question 4 which follows.

8/2

Question 4:

How many segments does each volume contain?

(A) 5

(B) 10

(C) 18

(D) 90

I

This choice is correct.

Proceed to page 3, section 1.5

The REM is the half page to which the program control refers you. What you are reading at this moment is a REM. In the study guide there are from 2 to 4 REMS for each question, and each punch card handles many questions. Therefore, this choice is not correct.

Return to page 12/2 and try question 5 again.

10/1

This choice is correct.

Now proceed to question 3 which follows:

Question 3

How many volumes is the study guide composed of?

(A) 5 (B) 10 (C) 18 (D) 90

10/2

The number of segments in a volume is discussed on page 1. If you reread that page, section 1.1, you will find that the correct number of segments is not the one you have chosen.

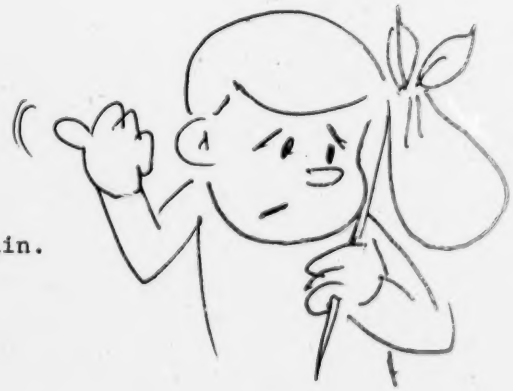
Return to page 8/2 and try question 4 again.

I

The punch card is the rectangular card in which you have punched choice B on line 2. Since it is not the book you are now reading, this choice is incorrect.

Return to page 7/1 and try question 2 again.

Since the course contains 18 volumes, and each volume contains 5 segments, you should be able to find out how many segments the entire course contains. Since the guide tells you that you will return the punch card to the instructor at the end of each segment, and that you will insert a new card into the program control device for each segment, it follows that you will need a card for each segment. Therefore, this choice is not correct.



Return to page 15/2 and try question 6 again.

12
1

According to section 1.1 on page 1 of the study guide, each volume contains 5 segments. Therefore, this choice is correct.

Proceed to question 5 which follows.

12
2

Question 5:

You will need a new punch card for each _____.

Which word should be inserted in the blank?

(A) segment

(B) question

(C) REM

(D) M.B.O.

The number of volumes in the course is discussed on page 1 of the study guide. the study guide covers the entire course. Therefore, this choice is not correct.

Return to page 10/1 and try question 3 again.

The answer matrix is actually a code to the position of the REM's in the study guide, since the pages have been scrambled. The answer matrix is placed in the program control device, and the only way you can get to read any part of it, is to punch a hole with your stylus.

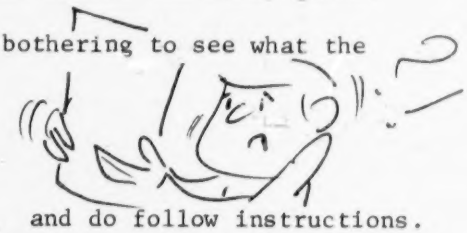
This choice is not correct.

Return to page 7/1 and try question 2 again.

$\frac{14}{1}$

You have done some correct arithmetic, but I'm afraid you forgot the meaning of the word "product." The product of two numbers is the result of multiplying them.

Return to page 3, section 1.4, and please follow the study guide. You went ahead and chose an answer without bothering to see what the guide had to say.



Remember, return to section 1.4, page 3, and do follow instructions.

$\frac{14}{2}$

An M.B.O. is one of the important points which you will have learned as a result of studying this course. The meaning of "M.B.O." was discussed on page 4. It is true that the questions you meet are related to M.B.O.'s, but you do not use a punch card for them.

This choice is not correct.

Return to page $\frac{12}{2}$ and try question 5 again.

On page 3, in section 1.5, you are told that you will return your punch card to the instructor at the end of the segment. On page 4, you are instructed to insert a punch card into the program control device before starting a segment. Therefore, you need a punch card for every individual segment.

This choice is correct.

Now proceed to question 6 which follows.

Question 6

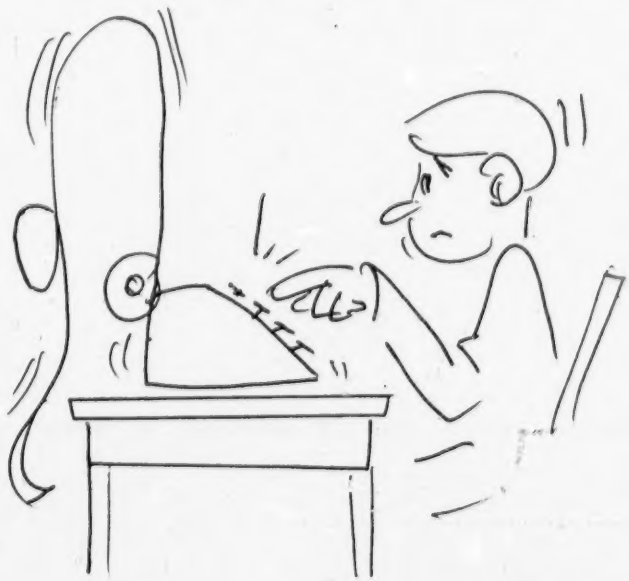
If we assume that no punch cards will be damaged, how many will a student require for the entire course?

(A) 5

(B) 10

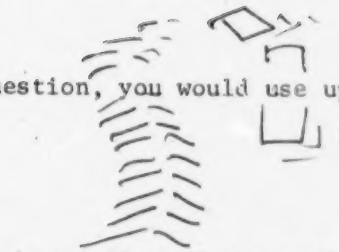
(C) 18

(D) 90



16
1

If you needed a new punch card for each question, you would use up an extremely large number of cards.



Each question is answered on the proper numbered line of the card.

Return to page 12/2 and try question 5 again.

16
2

An M.B.O. is one of the important points which you will have learned as a result of studying this course. It is not the name of the book.

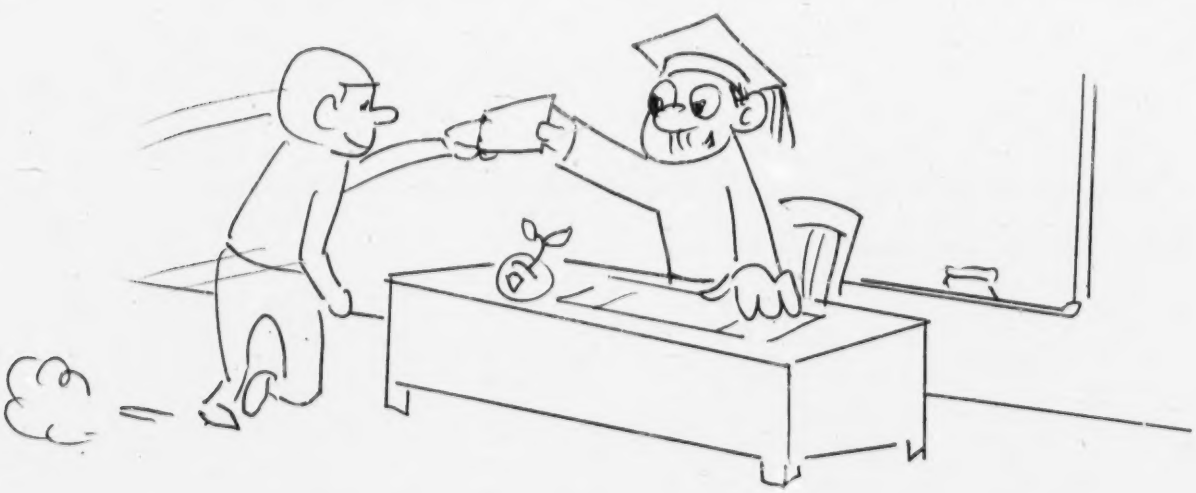
Return to page 7/1 and try question 2 again.

Since each segment calls for a new punch card, and there are 5 segments to a volume, a student needs 5 cards for each volume. There are 18 volumes, hence he needs 90 cards.

Therefore, this choice is correct.

You have now completed this segment. Before going on to Segment 2, be sure you have understood exactly what you are supposed to do. If you have punched more than one hole on a line of the punch card, except for line 1 where you should have punched 2, you have made an error. That is perfectly all right, provided that you learn from it. If you punched any hole without meaning to, you will have to practice being more careful.

When you are ready, ask your instructor for the answer matrix for Segment 2 and for a fresh punch card, and go ahead.



$\frac{18}{1}$

Volume 1 Segment 2 begins here:

Obtain a PUNCH CARD from your instructor. In addition to the other identifying information that must be furnished by you, you are asked to punch out the following:

COLUMNS	48	and	50	0 2	(Sequence Number)
	54	and	56	0 4	(Type of Punch Card)
	60	and	62	0 1	(Volume Number)
	66	and	68	0 2	(Segment Number)

PRELIMINARY NOTES

Some of the streets of a certain city are numbered consecutively from 1 to 10. Let's assume that you happen to live on street number 3, while a friend of yours lives on street number 7. If you walk from your house to your friend's house, you will pass streets number 4, 5, and 6. Now each of these streets is a place. We say, in mathematics, that with every number we can associate a point (place) on a number line. In this segment, this simple, but very important idea, will be developed and studied in more detail.

Your READING ASSIGNMENT for this Segment is pp. 1 - 5.

You will now be asked a series of questions to draw your attention to the more important points.

$\frac{18}{2}$

Recognize which of the following numbers are natural numbers.

$$0 \ , \ 3 \ , \ \frac{1}{8} \qquad 1\frac{1}{8} \ , \ \frac{8}{2}$$

Choose the letter which has only natural numbers, selected from the above set, written next to it.

(A) 0 , 3

(B) $1\frac{1}{8}$, 3 , $\frac{8}{2}$

(C) 3 , $\frac{8}{2}$

(D) 0 , 3 , $\frac{8}{2}$

Very good.

Whole numbers are the natural numbers with one addition. Zero is not a natural number, but it is a whole number. The number

$$\frac{2}{1} = 2 \text{ , and } \frac{3}{1} = 3$$

Thus, any natural number written with a denominator of 1 is a whole number.

Finally, $\frac{1.1}{1.1} = 1$ since any number, other than zero, divided by itself is equal to one.

Please go on to question 4 , below.

Question 4

Apply the proper principle and determine which of the following statements are true or false. Select the letter which labels the correct combination of truth values for your answer.

I. Every natural number can be written as a fraction.

II. There is a largest natural number.

III. Every fraction names a whole number.

(A) I is true
II is true
III is true

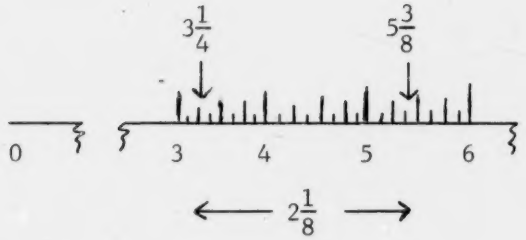
(B) I is false
II is false
III is true

(C) I is true
II is false
III is false

(D) I is true
II is false
III is true

$\frac{20}{1}$

Your answer is correct.



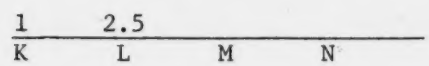
Please go on to question 8 below.

$\frac{20}{2}$

Question 8

Apply the principle and select the letter which correctly answers this question:

Points K , L , M , N are equidistant on the number line drawn below: what is the coordinate of N ?



(A) 5.0

(B) 10.0

(C) 7.5

(D) 5.5

$\frac{21}{1}$

Sorry, but your choice is not correct.

Counting numbers and natural numbers are the same.

Zero is not a counting number.

Please return to page $\frac{33}{2}$ and try question 2 again.

$\frac{21}{2}$

Did you mark of " tenths " on your number line?

Sorry, your answer is incorrect.

Please go back to page $\frac{35}{2}$ and try question 6 again.

I

$\frac{22}{1}$

If QR is twice as long PQ , we must first find the length of PQ. By subtracting P's coordinate from that of Q , we find PQ to be 4.75 units long. The QR is twice that, or

2 (4.75) or 9.50 units long.

Thus, the coordinate of R is found by adding:

	9.25	(the coordinate of Q)
+	9.50	(the distance from Q to R)
	18.75	the coordinate of point R.

Your answer is correct.

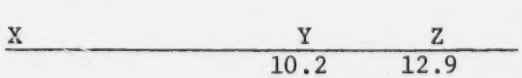
Please go on to question 10 , below.

$\frac{22}{2}$

Question 10

Apply the proper principle and select the letter which correctly answers this question:

What is the coordinate of point X on the number line drawn below if YZ is $\frac{1}{3}$ as long as XY ?



(A) 2.1

(B) 8.1

(C) 4.3

(D) 3.4

The natural numbers are 1 , 2 , 3 . . . . and so on.

Zero is not a natural number.

Please return to page 18/2 and try question 1 again.

We don't agree.

Is there a largest natural number? Suppose you write down the largest natural number you can:

We can always make it larger by adding 1 to it.

Thus, there is no largest natural number.

Please return to page 19/2 and try question 4 again.

$\frac{24}{1}$

Numbers such as

$\frac{1}{2}$ and $\frac{1}{3}$ are called fractions.

Fractions are not natural numbers.

Please return to page $\frac{36}{2}$ and try question 3 again.

$\frac{24}{2}$

Did you forget the meaning of " between " ?

Between means not including the end points.

For example,

T , U , V , and W are between S and X , or in other words, there are four letters between S and X in the alphabet.

Return to page $\frac{38}{2}$ and try question 11 again.

Did you draw a number line to help you with this problem?

Find point 7 , now count 4 units to the left, and then another half unit to the left.

Please return to page $\frac{31}{2}$ and try question 5 again.

Draw a number line and mark off segments of $\frac{1}{4}$ units. Number all the integral values, then find

$2\frac{3}{4}$ and $8\frac{1}{2}$ on that line.

What are the natural numbers between them?

Please return to page $\frac{43}{2}$ and try question 12 again.

$\frac{26}{1}$

The number $1\frac{2}{3}$ is a mixed number, not a counting number. Mixed numbers as well as zero and decimal numbers are not considered " natural " numbers or counting numbers.

Please return to page $\frac{33}{2}$ and try question 2 again.

$\frac{26}{2}$

It's very easy to be careless!

Re-read the question. Does it say that R is double Q ? Or, does it rather say that QR is twice as long as PQ ?

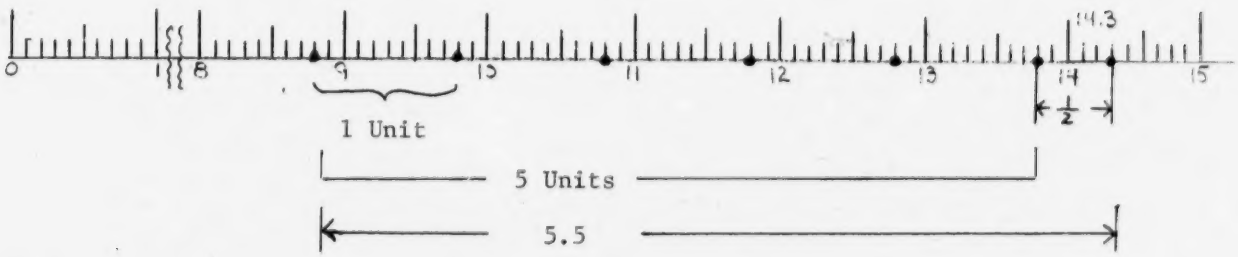
How would you determine how long PQ is?

Please return to page $\frac{40}{2}$ and try question 9 again.

After drawing the number line, we can mark off divisions of " tenths ".

Now we can begin at 8.8 and count 5.5 places.

Your answer is correct.



Please go on to question 7 below.

Question 7

Apply the proper principle and select the letter which correctly answers the question:

Two points $3\frac{1}{4}$ and $5\frac{3}{8}$ are chosen on a one-foot ruler; how far apart are these points?

(A) $2\frac{1}{8}$

(B) $2\frac{1}{4}$

(C) $1\frac{1}{8}$

(D) $12\frac{5}{8}$

$\frac{28}{1}$

We don't agree.

The number $1\frac{1}{8}$ is called a mixed number. It consists of a natural number and a fraction.

Please return to page $\frac{18}{2}$ and try question 1 again.

$\frac{28}{2}$

Sorry, we don't agree.

Can you represent the fraction one-third as a whole number?

Please consider your choice.

Return to page $\frac{19}{2}$ and try question 4 again.

$\frac{3}{1}$ is considered a whole number since it is written with a denominator of 1 .

Any number divided by one equates itself. However, if the numerator is one such as the number $\frac{1}{2}$, it is a fraction; not a natural number.

Please return to page $\frac{36}{2}$ and try question 3 again.

It's easy to be careless.

How large is the interval between points K and L ?

Please return to page $\frac{20}{2}$ and try question 8 again.

$\frac{30}{1}$

Good! Your answer is correct.

When we are asked for numbers " between " two other numbers, the end-points are not considered.

Please go on to question 13 below.

$\frac{30}{2}$

Question 13

Apply the proper principle and find how many natural numbers there are between .7 and 8.7 . Select the letter which labels the correct statement.

(A) 8

(B) 7

(C) 6

(D) 9

You correctly omitted 0 and $1\frac{2}{3}$ which are not counting numbers. However, you included decimal numbers, which also are not " natural " or counting numbers.

Please return to page 33/2 and try this question again.

Your answer is correct.

Please go on to question 5 which follows:

Question 5

Draw a segment of the number line for reference and label it appropriately. Apply the proper principle and find the coordinate of the point that is $4\frac{1}{2}$ units to the left of point 7 .

Select the letter which labels the correct statement.

(A) $3\frac{1}{2}$ (C) $1\frac{1}{2}$

(B) $2\frac{1}{2}$ (D) 3

I

$\frac{32}{1}$

What interval did you use on your number line?

Did you make each point $\frac{1}{8}$ of a unit away from the previous one?

$3\frac{1}{4}$ then, can be considered $3\frac{2}{8}$

Please return to page $\frac{27}{2}$ and try question 7 again.

$\frac{32}{2}$

It's very easy to be careless!

Please re-read the directions in question 10.
How long is segment YZ? What relationship does this have in our problem? If you find YZ, then find the length of XY. Then can you see how to find the coordinate of X?

Please return to page $\frac{22}{2}$ and try question 10 again.

This is correct.

Although $\frac{8}{2}$ looks like a fraction, it actually is a numeral for 4 and is, therefore, a natural number. The natural numbers are the " counting numbers ".

Please go on to question 2 below.

Question 2

Recognize which of the following numbers are counting numbers.

$$0 \ , \ 1 \ , \ 1.1 \ , \ 1.2 \ , \ 2 \ , \ 1\frac{2}{3}$$

Select the letter which has only counting numbers selected from the above set written next to it.

(A) 1 , 1.1 , 1.2 , 2

(B) 0 , 1 , 2

(C) 1 , $1\frac{2}{3}$, 2

(D) None of these other choices are correct.

$\frac{34}{1}$

We disagree.

One of the letters does have the correct answer next to it.

Please reconsider your choice.

Please return to page $\frac{36}{2}$ and try question 3 again.

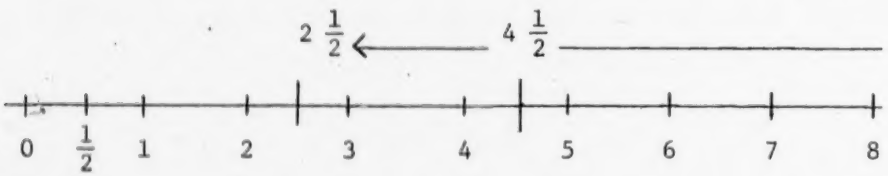
$\frac{34}{2}$

Please check your arithmetic!

Segment PQ's length can be determined by subtracting the coordinate of P from that of Q.

Please return to page $\frac{40}{9}$ and try question 9 again.

After drawing a number line, we can locate the point, 7 , and then count four units to the left, arriving at 3 . Now go one-half a unit farther to the left, $2\frac{1}{2}$.



Your answer is correct.

Please go on to question 6 below.

Question 6

Draw a segment of the number line and label it appropriately. Apply the proper principle and find the coordinate of a point that is 5.5 units to the right of point 8.8 . Select the letter which labels the correct statement.

(A) 15.3

(B) 14.3

(C) 3.3

(D) 13.3

Zero, the mixed number, $1\frac{2}{3}$, and the decimals, 1.1 and 1, 2 are not natural numbers.

Since each of the other answer choices contained at least one of these along with the counting numbers, your answer choice is correct.

Please go on to question 3 below.

Question 3

Recognize which of the following numbers are whole numbers.

$$0 \ , \ \frac{2}{1} \ , \ \frac{1}{2} \ , \ \frac{3}{1} \ , \ \frac{1}{3} \ , \ \frac{1.1}{1.1}$$

Select the letter which has only whole numbers selected from the above set, written next to it.

(A) $0 \ , \ \frac{2}{1} \ , \ \frac{3}{1} \ , \ \frac{1.1}{1.1}$

(B) $\frac{2}{1} \ , \ \frac{1}{2} \ , \ \frac{3}{1} \ , \ \frac{1}{3}$

(C) $0 \ , \ \frac{1}{2} \ , \ \frac{3}{1}$

(D) None of these other choices are correct.

Sorry, we don't agree.

Did you count <u>from</u> the point $3\frac{1}{4}$ to the $5\frac{3}{8}$?

It seems that you counted $3\frac{1}{4}$ <u>beyond</u> $5\frac{5}{8}$.

Please return to page $\frac{27}{2}$ and try question 7 again.

It's very easy to be careless.

The fraction $\frac{22}{7}$ does not symbolize the interval between 7 and 22 .

What does a fraction sygnify?

Please return to page $\frac{58}{2}$ and try question 15 again.

38
1

Segment YZ is 2.7 units in length. Segment XY is three times that; so, XY is 8.1 units in length. Since X is to the left of Y on the number line, it's coordinate must be less. Therefore, we must subtract the length of XY from the coordinate of Y to find the coordinate of X.

Since Y is 10.2 when we subtract the length of XY , the coordinate of X is 2.1 .

$$\begin{array}{r} 10.2 \\ -\ 8.1 \\ \hline 2.1 \end{array}$$

Your answer is correct.

Please go on to question 11 below.

38
2

Question 11

Apply the proper principle and find how many natural numbers there are between 10 and 15 . Select the letter which labels the correct answer.

(A) 4

(B) 5

(C) 6

(D) An infinite number.

I

Good! Your answer is correct.

The first natural number after .7 is 1. Before you reach 8.7 , you have counted through 1 , 2 , 3 , 4 , 5 , 6 , 7 , and 8. Thus, there are eight natural numbers between .7 and 8.7

Please go on to question 14 below.

Question 14

Apply the proper principle and find a number between 1 , 4 and 1.5 . Select the letter which labels the correct statement.

(A) 2.9

(B) 1.14

(C) 1.54

(D) 1.45

$\frac{40}{1}$

You can find the distance between K and L by subtracting their coordinates. Since each of the points are equidistant, the coordinate of N is

$$2.5 + 1.5 + 1.5$$

Coordinate of N is 5.5 .

Your answer is correct.

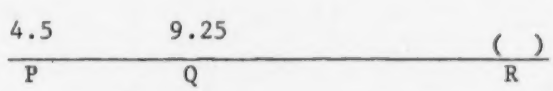
Please go on to question 9 below.

$\frac{40}{2}$

Question 9

Apply the proper principle and select the letter which correctly answers this question:

What is the coordinate of point R on the number line drawn below if QR is twice as long as PQ ?



(A) 27.75

(B) 13.75

(C) 18.75

(D) 18.50

I

The improper fraction $\frac{22}{7}$ can be expressed as a mixed number. A mixed number is made up of the natural number which is the largest possible quotient and a remainder. As a mixed number $\frac{22}{7}$ is expressed as $3\frac{1}{7}$

Please return to page $\frac{58}{2}$ and try question 15 again.

We're sorry, but we disagree.

$\frac{3}{4}$ is not between 1 and $\frac{3}{2}$. Did you draw a number line to help you?

Please return to page $\frac{52}{2}$ and try question 19 again.

$\frac{42}{1}$

Between $1\frac{1}{4}$ and 2 exclusive, there are two marks; one at $1\frac{1}{2}$ the other at $1\frac{3}{4}$. Between 2 and 3 inclusive, there are five marks. There are two more at $3\frac{1}{4}$ and $3\frac{1}{2}$. Thus, there are

$$2 + 5 + 2$$

or nine marks in all.

On looking at it in another way,

$$1\frac{1}{4} = \frac{5}{4}$$

and

$$3\frac{3}{4} = \frac{15}{4}$$

Between 5 fourths and 15 fourths there are 9 fourths.

Your answer is correct.

Please go on to question 17 below.

$\frac{42}{2}$

Question 17

Apply the proper principle and find how many whole numbers there are between $\frac{72}{6}$ and $\frac{6}{2}$ inclusive, that are also divisible by 3.

Select the letter which labels the correct answer.

(A) 2

(B) 3

(C) 4

(D) None of these.

Right! There are four natural numbers between 10 and 15 .

The idea of "between" means that the end points are not to be part of the segment.

Thus, we only consider the numbers,

11, 12, 13, and 14

Please go on to question 12 below.

Question 12

Apply the proper principle and find how many natural numbers there are between

$2\frac{3}{4}$ and $8\frac{1}{2}$.

Select the letter which labels the correct answer.

(A) 8

(B) 7

(C) 6

(D) 5

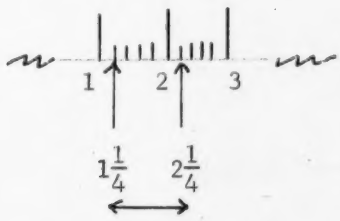
$\frac{44}{1}$

It's very easy to be careless. You were not asked for the sum of the coordinates, you were asked to find a number between 1.4 and 1.5 .

Please return to page $\frac{39}{2}$ and try question 14 again.

$\frac{44}{2}$

Sorry, we disagree. Did you sketch a number line to help you with this problem?



Now we're assuming that you can find the midpoint.

Please return to page $\frac{56}{2}$ and try question 18 again.

It seems as though $\frac{1}{5}$ would be the midpoint between $\frac{1}{4}$ and $\frac{1}{6}$ because 5 is the midpoint between 4 and 6 .

However, it is not true. The point midway between two points could be considered as the "average" of the two points.

Please return to page $\frac{62}{2}$ and try question 20 again.

This is only one of three choices which you were offered which have the value 14 .

Therefore, this choice is not correct.

Return to page $\frac{69}{1}$ and try question 1 again.

I

46/1

It's very easy to be careless! The information that QR is twice as long as PQ does NOT mean add the coefficients of P and Q to find R .

Please return to page 40/2 and try question 9 again.

46/2

Did you draw a number line to help you with this question? Find the point corresponding to .7 ; find the spot that corresponds to 8.7 and then count the number of natural numbers that fall between your two points.

Please return to page 30/2 and try question 13 again.

I

Since the fraction

$\frac{21}{7}$ equals 3 ,

and

$\frac{28}{7}$ equals 4 ,

then the fraction

$\frac{22}{7}$

is between the natural numbers

3 and 4 .

Your answer is correct.

Please go on to question 16 below.

Question 16

Apply the proper principle and select the letter which correctly answers the following question:

A 6-inch ruler has a mark for every one-quarter of an inch. How many of these marks are there between $1\frac{1}{4}$ and $3\frac{3}{4}$?

(A) 12

(B) 11

(C) 10

(D) 9

48/1

It's easy to be careless. Did you draw a number line to assist you? Divide your number line into fourths.

Please return to page 52/2 and try question 19 again.

48/2

What is the value of 4.5 + 3 ? And what is the value of 3 + 4.5 ? If you still get his choice, you must have made a mistake in addition since this choice is not correct.

Please return to page 60/2 and try question 2 again.

Good guess! If the question had asked "Find how many numbers there are between 10 and 15 " you would have been correct. You overlooked the adjective, "Natural" in the description. The natural numbers between 10 and 15 can be counted and thus are not Infinite.

Please return to page $\frac{38}{2}$ and try question 11 again.

Let us ask you a similar question.

Is \$1.14 between \$1.40 and \$1.50 ?

Please return to page $\frac{39}{2}$ and try question 14 again.

I

$\frac{50}{1}$

Sorry, we don't agree. The number two can be expressed as $\frac{14}{7}$ and the number 3 as $\frac{21}{7}$. The number $\frac{22}{7}$ is not between those two.

Please return to page $\frac{58}{2}$ and try question 15 again.

$\frac{50}{2}$

Sorry, we don't agree. Did you forget the meaning of "inclusive"? Suppose you correct the given fraction to whole numbers and then list the numbers between them on the number line. The re-read the rest of the question before you make your answer choice.

Please return to page $\frac{42}{2}$ and try question 17 again.

I

51/1

If you are asked to tally (count) a number of items, a convenient way of doing it is to put down a mark for each item.
To make it easier to total, it is customary to place every fifth mark across the other four. This choice then offers the number five plus five plus four, which is certainly 14. Then this choice is not correct because its value is the same as two other choices.

Please return to page 69/1 and try question 1 again.

51/2

You are probably used to doing subtractions with one number written under the other. If you write it this way:

$$\begin{array}{r} 5.2 \\ \underline{-1.7} \end{array}$$

Do you now see your error? This choice is not correct.

Please return to page 65/2 and try question 3 again.

I

$\frac{52}{1}$

A synonym for "midway" is halfway. For example, midway between one and two is the point $1\frac{1}{2}$. Now another common method used to find the midpoint, or average of two numbers is to add them and divide by two. The sum of $1\frac{1}{4}$ and $2\frac{1}{2}$ is $3\frac{2}{4}$ or $3\frac{1}{2}$. One-half of $3\frac{1}{2}$ can be found by converting the mixed number to an improper fraction.

$3\frac{1}{2}$ equals $\frac{7}{2}$.

$\frac{1}{2}$ of $\frac{7}{2}$ is $\frac{7}{4}$.

This can be reformed as a mixed number $1\frac{3}{4}$

Your answer is correct.

Please go on to question 19 below.

$\frac{52}{2}$

Question 19

Apply the proper principle and find the coordinate of the point that lies midway between 1 and $\frac{3}{2}$. Select the letter which labels the correct statement.

(A) $\frac{3}{4}$

(B) $\frac{6}{5}$

(C) $\frac{4}{2}$

(D) $\frac{5}{4}$

I

$\frac{53}{1}$

Let us ask you a similar question.

Is \$1.54 between \$1.40 and \$1.50 ?

Please return to page $\frac{39}{2}$ and try question 14 again.

$\frac{53}{2}$

Sorry, we disagree!

Make a sketch of such a ruler to help you.

Return to page $\frac{47}{2}$ and try question 16 again.

I

$\frac{54}{1}$

It's easy to be careless. To find an average of two points or the midpoint, you take <u>half</u> the sum of their coordinates.

Please return to page $\frac{62}{2}$ and try question 20 again.

$\frac{54}{2}$

Since the sum of any two numbers is a number, both of the additions are possible.

Therefore, this choice is not correct.

Return to page $\frac{60}{2}$ and try question 2 again.

The proper way to check a division example is to multiply the quotient by the divisor to see if you get the dividend. Using this test, does

$$7 = 0 \times 7 \ ?$$

It does not. Therefore, this choice is not correct.

Please return to page 80/2 and try question 5 again.

What did you get for the value of R ? If you had trouble dividing 2 by .2 , you should remember that it is necessary to move the decimal point in the divisor to the right of the number. Since that moves it one place to the right, you must then move the decimal point in the dividend one place to the right as well. Since the dividend was 2 .

What did you get when you moved the decimal point?

$$\text{e.g.} \quad \frac{2}{.2} = \frac{2.0}{.2} = \frac{20}{2}$$

Please return to page 90/2 and try question 7 again.

I

$\frac{56}{1}$

Congratulations! Your answer shows that you recognized that the word "inclusive" means that the end points of a segment are included in the interval. Let us review this problem.

$\frac{72}{6}$ is the same as 12

$\frac{6}{2}$ is the same as 3

The whole numbers between 12 and 3 inclusive are namely,

12, 11, 10, 9, 8, 7, 6, 5, 4, 3 .

Of these 12, 9, 6, and 3 are divisible by 3 .

Please continue with question 18 below.

$\frac{56}{2}$

Question 18

Apply the proper principle and find the coordinate of the point midway between $1\frac{1}{4}$ and $2\frac{1}{4}$.

Select the letter which labels the correct number.

(A) $1\frac{2}{4}$ (C) $1\frac{1}{3}$

(B) $1\frac{3}{4}$ (D) 1.2

$\frac{57}{1}$

Sorry, we don't agree. When two fractions are added, we do NOT merely combine their denominators.

Please return to page $\frac{62}{2}$ and select another answer for question 20 .

$\frac{57}{2}$

Since this choice offers two different subtractions, and one of them is not possible to do, this choice is not correct.

Return to page $\frac{71}{2}$ and try question 4 again.

I

$\frac{58}{1}$

Your answer is correct. Congratulations!

You can always find a number between two other numbers by finding their average, that is $\frac{1}{2}$ of their sum.

Please go on to question 15 below.

$\frac{58}{1}$

Question 15

Apply the proper principle and find between what two successive natural numbers $\frac{22}{7}$ lies. Select the letter which labels the correct answer.

(A) 7 and 22

(B) 3 and 4

(C) 3 and $3\frac{1}{7}$

(D) 2 and 3

I

Your answer choice indicated that $\frac{4}{2}$ is midway between 1 and $\frac{3}{2}$, did you check your statement? Did you draw a number line to help you?

Please return to page $\frac{52}{2}$ and try question 19 again.

Writing the example in the way you are used to seeing it:

$$\begin{array}{r} 1.7 \\ -5.2 \end{array}$$

You should recognize that the larger number is being taken away from the smaller, which is impossible for you to do (at this time).

Therefore, this choice is not correct.

Return to page $\frac{65}{2}$ and try question 3 again.

I

60
1

In Roman numerals this number has the value 10 (for X) plus 5 (for V) plus 1 (for I). Its value is, therefore, 16, which is not the same as the value of the other choices.
Therefore, this choice is correct.

Proceed to question 2 below.

60
2

Question 2

If P is 4.5 + 3 , and Q is 3 + 4.5 , which statement do you recognize is correct?

(A) P is larger than Q

(B) P is smaller than Q

(C) P has the same value as Q

(D) these additions are meaningless

This choice says that 2 divided by 4 equals 2 . The best way to check a division is by multiplying quotient by divisor to see if you get the dividend. But 2 does not equal 4 times 2 . Therefore, this choice is not correct.

Return to page 80/2 and try question 5 again.

Did you calculate the value of each of the letters? You have a mistake in arithmetic somewhere, since this choice is not correct.

Return to page 90/2 and try question 7 again.

I

$\frac{62}{1}$

Congratulations! Your answer is correct. If you divide your number line into fourths, 1 is the fourth mark from zero,

$\frac{3}{2}$ is the sixth mark from zero.

The point midway between these two is the fifth mark which corresponds to $\frac{5}{4}$.

Please go on to question 20 below.

$\frac{62}{2}$

Question 20

Apply the proper principle and find the coordinate of the point that lies midway between $\frac{1}{4}$ and $\frac{1}{6}$

(A) $\frac{1}{5}$ (C) $\frac{1}{10}$

(B) $1\frac{5}{12}$ (D) $\frac{5}{24}$

63/1

Since this choice offers two different divisions, where the order of the numbers is changed, the results are not equal. In general, the division of any two numbers is possible, but the result depends on the order of the numbers. In any event, this choice is incorrect.

Return to page 71/2 and try question 4 again.

63/2

Since the sum of two numbers does not depend on the order of addition, this statement is correct. But you were asked for the one which is not correct.

Therefore, this is not the correct choice.

Return to page 85/2 and try question 6 again.

I

$\frac{64}{1}$

Since 5 divided by 3 is $1\frac{2}{3}$ which is not larger than 2, this choice is not correct.

Return to page $\frac{73}{2}$ and try question 8 again.

$\frac{64}{2}$

In adding two fractions, the result is larger than either one. However, in multiplying two fractions (proper fractions), the reverse is true, since part of a quantity is smaller than the original. Then it is impossible for S and T to have the same value, and this choice is not correct.

Return to page $\frac{95}{2}$ and try question 10 again.

Since the value of P is 7.5 , and the value of Q is 7.5, this is correct.

Proceed to question 3 below.

Question 3

Which choice do you recognize is correct?

(A) $5.2 - 1.7 = 4.5$ (C) $1.7 - 5.2 = 3.5$

(B) $1.7 - 5.2 = 4.5$ (D) $5.2 - 1.7 = 3.5$

I

$\frac{66}{1}$

A simple rule which you should know is that if two fractions have the same numerator, the one with the larger denominator is the smaller fraction.

Therefore, this choice is not correct.

Return to page $\frac{91}{2}$ and try question 9 again.

$\frac{66}{2}$

As pointed out in the Preliminary Notes to this segment, you may not divide by zero. Therefore, 5 divided by zero is not a number, and it cannot be compared in size to a number. Then this choice is not correct.

Return to page $\frac{77}{2}$ and try question 11 again.

The midpoint between $\frac{1}{4}$ and $\frac{1}{6}$ can be determined by averaging the coordinates $\frac{1}{4}$ and $\frac{1}{6}$

The sum of $\frac{1}{4}$ and $\frac{1}{6}$ is $\frac{5}{12}$

Next, we take one-half of this, thus, the coordinates of the midpoint are

$$\frac{5}{24}$$

Your answer is correct.

You have now finished this SEGMENT. Hand in the PUNCH CARD.

You should have entered in your NOTEBOOK the following:

1. counting numbers are also called natural or whole numbers.
2. " between " means that the end points of the segment are not included.
3. " inclusive " means the end points are also part of the segment.
4. including means that only one end point is included.

You should now be able to complete the following from your homework:

pg. 3 - 4 questions 1 - 10

pg. 17 questions 1 - 12

Volume 1 Segment 3 begins here:

Obtain a Punch Card from your instructor. In addition to the other identifying information that must be furnished by you, you are asked to punch out the following.

COLUMNS	48	and	50	0 3	(Sequence Number)
	54	and	56	0 4	(Type of Punch Card)
	60	and	62	0 1	(Volume Number)
	66	and	68	0 3	(Segment Number)

Preliminary Notes:

The four fundamental operations of arithmetic (addition, subtraction, multiplication and division) are familiar to you. However, there are certain facts that we ought to review, even though you may feel that they are obvious.

1. When any two numbers are added, the result is a number, and there is only one result.
2. When any two numbers are multiplied, the result is a number, and there is only one result.
3. When one number is subtracted from another and there is a result, there is only one result.
4. When one number is divided by another and there is a result, there is only one result.

You understand, of course, that a number may be large or small, integral (whole number) or fractional, or even zero.

Now the first two statements guarantee that addition of two numbers or multiplication of two numbers is always possible and give a single result. Statement number 3 suggests that subtraction is not always possible, and this is true.

continued

The number being subtracted must not be larger than the number we are subtracting it from. May the two be equal ? Of course, since a number subtracted from itself gives zero.

Division also does not guarantee a result. When may a division be impossible ? The answer is that any division is possible, except ... , but we'll get to that. You know that

$$6 \div 2 = 3, \text{ and}$$

$$2 \div 6 = \frac{1}{3} .$$

The division of one number by another, in that order, gives a single result. In a different order, it would be a different divison. Now for the exception: division by zero is impossible! Why is that ?

Let's look at $6 \div 3$.

We say the result is 2 because

$$6 = 3 \times 2 .$$

Then, if

$$6 \div 0 = (\text{a number})$$

It must follow that

$$6 = 0 \times (\text{a number})$$

But zero times any number equals zero. Therefore, YOU MAY NOT DIVIDE BY ZERO.

Now turn to your reading assignment for this segment: pages 5 - 8.

You will now be asked a series of questions to draw your attention to the more important points.

Question 1

Which choice do you recognize is a numerical expression for a number different from the others ?

A. 14
B. XVI
C. fourteen
D. ~~IIII IIII IIII~~

I

$\frac{70}{1}$

Since the product of two numbers does not depend on the order of multiplication, this statement is correct; but you were asked for the one which is not correct. Therefore, this is not the correct choice.

Please return to page $\frac{85}{2}$ and try question 6 again.

$\frac{70}{2}$

In adding two fractions, the result is larger than either one. However, in multiplying two proper fractions, the reverse is true, since part of a quantity is smaller than the original.

Then: S is larger than $\frac{1}{2}$,,

while T is smaller than $\frac{1}{2}$.

Therefore, this choice is not correct.

Please return to page $\frac{95}{2}$ and try question 10 again.

Writing the example in the more usual form, we have:

$$\begin{array}{r} 5.2 \\ \underline{-1.7} \end{array}$$

Performing the subtraction and being careful with our arithmetic, we get the result you chose.

Therefore, this choice is correct.

Please proceed to question 4 which follows.

71
1

Question 4

Which choice do you recognize is correct?

(A) $4 - 3 = 3 - 4$

(B) $4 \div 3 = 3 \div 4$

(C) $4 \times 3 = 3 \times 4$

(D) None of these.

I

72/1

If you add these numbers keeping the decimal points in line, the result is

1.2

which is certainly not smaller than

1

Therefore, this choice is not correct.

Please return to page 83/2 and try question 12 again.

72/2

This choice says that the value on the right side is

3.5

Is that the value you got for the left side? The left side said that you were to divide the total of

3 + 2 by 2

If you insist on dividing before you add, you must divide <u>each</u> number by

2

This choice is not correct.

Please return to page 98/2 and try question 13 again.

The value of

$P = 8 + 2$

is 10

The value of $Q = 5 \times 2$

is 10

The value of $R = 2 \div .2$

can be calculated as follows:

$.2 \overline{)2}$

$.2\overline{).20}$. { shifting the decimal point one place to the right.

Then the value of R is also 10.

This choice is correct.

Please proceed to question 8 below.

Question 8

Which one of the statements do you recognize as correct ?

(A) $5 \div 3 > 2$

(B) $5 \div 3 < 2$

(C) $5 - 3 < 2$

(D) $5 \times 3 < 2$

$\frac{74}{1}$

This choice says that 0

divided by 7

equals 7

The best way to check a division is by multiplying quotient by divisor to see if you get the dividend. But

0

does not equal 7×7

Therefore, this choice is not correct.

Please return to page $\frac{80}{2}$ and try question 5 again.

$\frac{74}{2}$

Since $6.2 \times 70 = 434$

and $350 + 84 = 434$

this choice does not make the inequality true. Therefore, this choice is not correct.

Please return to page $\frac{104}{2}$ and try question 14 again.

Since the same number, 5 , is divided by two different numbers; the results must have different values. Therefore, this choice is not correct.

Please return to page 91/2 and try question 9 again.

Since the number 7 + 3

equals 10

and 10

divided by 2

equals 5

which is not larger than

6

this choice is not correct.

Please return to page 107/2 and try question 16 again.

I

$\frac{76}{1}$

Since one of the other choices is correct, this is not.

Please return to page $\frac{71}{2}$ and try question 4 again.

$\frac{76}{2}$

Since one of the other choices is correct, this is not.

Please return to page $\frac{85}{2}$ and try question 6 again.

I

In adding two fractions, the result is larger than either one. However, in multiplying two proper fractions, the reverse is true since part of a quantity is smaller than the original.

Then

S is larger than $\frac{1}{2}$,

while

T is smaller than $\frac{1}{2}$.

Therefore, this choice is correct.

Please proceed to question 11 below.

Question 11

Which choice do you recognize as correct ?

(A) $5 \div 0 > 5$

(B) $5 \div 0 = 5$

(C) $5 \div 0 < 5$

(D) None of these.

I

78/1

Since 5 - 3

is not smaller than 2

this choice is not correct.

Please return to page 73/2 and try question 8 again.

78/2

Since 5 x 1 = 5

and 5 + 2 = 7

which is not smaller than 4

this choice is not correct.

Please return to page 100/2 and try question 15 again.

You may not have been able to think of it, but there is more than just the number 1 which has this property.

Will this hint help? A number fits this property is a very familiar one, but one which you often do not think of as a number.

Please return to page 97/2 and try question 17 again.

The proper procedure is to find the value of the numerator first, and then divide. If you do this, you will find that the fraction on the left does not have the same value as the sum on the right. Therefore, this choice is not correct.

Did you decide to divide one of the numbers by 2 before adding? That is not correct. If you insist on dividing before performing the addition, you must divide <u>each</u> number by the 2 .

Please return to page 93/2 and try question 18 again.

I

80
1

Since the result in a multiplication does not depend on the order of the numbers, this choice is correct.

Please proceed to question 5 below.

80
2

Question 5

Which choice do you recognize as correct ?

(A) $7 \div 7 = 0$

(B) $2 \div 4 = 2$

(C) $0 \div 7 = 7$

(D) $2 \div 4 = .5$

I

Since 5×3

is not smaller than 2

this choice is not correct.

Please return to page 73/2 and try question 8 again.

Since $6.2 \times 70 = 434$

and $350 + 12 \times 7 = 350 + 84 = 434$

this choice does not make the inequality true.

Therefore, this choice is not correct.

Please return to page 104/2 and try question 14 again.

I

$\frac{82}{1}$

If one is subtracted from both the numerator and the denominator of a fraction, the resulting fraction does not have the same value as the original fraction.

e.g. $\frac{9}{3}$ (which equals 3)

would become $\frac{9 - 1}{3 - 1} = \frac{8}{2}$ (which equals 4)

Please return to page $\frac{91}{2}$ and try question 9 again.

$\frac{82}{2}$

Since the number 8 + 3 equals 11

and 11 divided by 2

is $5\frac{1}{2}$

which is not larger than 6

this choice is not correct.

Please return to page $\frac{107}{2}$ and try question 16 again.

Since none of the other choices is correct, this choice is correct.

Notice that 5 divided by 0 is not a number, and it cannot be compared to size to any number.

Please proceed to question 12 below.

Question 12

Perform the calculation to find which choice is correct.

(A) $.4 + .8 < 1$

(B) $25.47 \times 4 > 100$

(C) $4 + \frac{4}{3} \neq 4 \times \frac{4}{3}$

(D) $\frac{22}{7} + \frac{7}{11} = 2$

84/1

Have you found some numbers which have the property mentioned? When you multiply your number by itself, you are supposed to get the same number again. This choice says that you found many numbers like this. Since that is impossible, you have misunderstood the problem in some way.

This choice is not correct.

Please return to page 97/2 and try question 17 again.

84/2

If you find the value in the numerator first, you get 10 .

But 10 divided by 6 is less than 2 ;

it is certainly not larger that 5 .

Therefore, this choice is not correct. Note that it is important to add before doing the division.

Please return to page 102/2 and try question 19 again.

This choice says that

2 divided by 4 equals .5

The best way to check a division is by multiplying quotient by divisor to see if you get the dividend.

Does 2 equal 4 times .5 ? Yes, it does.

Therefore, this choice is correct.

Please proceed to question 6 below.

Question 6

Which statement do you recognize is NOT correct?

(A) $8.3 \div 2 = 2 \div 8.3$

(B) $8.3 \times 2 = 2 \times 8.3$

(C) $8.3 + 2 = 2 + 8.3$

(D) None of these.

$\frac{86}{1}$

Since one of the other choices is correct, this is not.

Please return to page $\frac{95}{2}$ and try question 10 again.

$\frac{86}{2}$

$$5 \times \frac{1}{2} = 2.5$$

and

$$2.5 + 2 = 4.5$$

which is not smaller than 4 .

Therefore, this choice is not correct.

Please return to page $\frac{100}{2}$ and try question 15 again.

This choice says that the value on the right side is 4 . Is that the value you got for the left side? The left side said that you were to divide the total of

$$3 + 2 \text{ by } 2$$

If you insist on dividing before you add, you must divide <u>each</u> number by 2 .

This choice is not correct.

Please return to page 98/2 and try question 13 again.

Since one of the other choices is correct, this is not.

Please return to page 93/2 and try question 18 again.

$\frac{88}{1}$

What number did you find? If you multiply your number by itself, do you get the number you started with as a result?

That is what this choice says, and it is not correct.

Please return to page $\frac{97}{2}$ and try question 17 again.

$\frac{88}{2}$

When two numbers are added, the result is larger than either one.

Therefore, $\frac{1}{2} + \frac{1}{2}$ cannot be smaller than $\frac{1}{2}$,

and this choice is not correct.

Please return to page $\frac{99}{2}$ and try question 20 again.

$$4 + \frac{4}{3} = \frac{12}{3} + \frac{4}{3}$$

since it is necessary to get a common denominator when adding fractions.

When multiplying fractions, the procedure is different:

$$\frac{4}{1} \times \frac{4}{3} = \frac{4 \times 4}{1 \times 3}$$

$$= \frac{16}{3}$$

Then the results are equal, therefore, this choice is not correct.

Please return to page $\frac{83}{2}$ and try question 13 again.

Since the number

9 + 3 equals 12

and

12 divided by 2 equals 6

which is not larger than

6

This choice is not correct.

Please return to page $\frac{107}{2}$ and try question 16 again.

Since this choice offers the same two numbers divided in different orders, the results are not equal.

Therefore, since we were asked for the statement which is not correct, this choice is the proper choice.

Please proceed to question 7 below.

Question 7

Consider the following numbers:

P is 8 + 2

Q is 5 x 2

R is 2 ÷ 2

Perform the calculations to find which statement is true.

(A) All three are equal

(B) Only P and Q are equal

(C) Only Q and R are equal

(D) Only P and R are equal

Since

5 divided by 3 is 1

which is smaller than 2 .

This choice is correct.

Proceed to question 9 below.

Question 9

Which statement do you recognize as correct?

(A) $\frac{5}{13} > \frac{5}{12}$

(B) $\frac{5}{13} = \frac{5}{12}$

(C) $\frac{5}{13} < \frac{5}{12}$

(D) $\frac{5}{13} = \frac{4}{12}$

$\frac{92}{1}$

Since one of the other choices is correct, this is not.

Return to page $\frac{98}{2}$ and try question 13 again.

$\frac{92}{2}$

In order to subtract fractions, it is necessary to find a common denominator. In this case the common denominator would be 6 .

Showing the work in horizontal form, which we usually use in algebra; we have:

$$\frac{5}{6} - \frac{1}{3} = \frac{5}{6} - \frac{2}{6} = \frac{3}{6} = \frac{1}{2}$$

Since this value is not smaller than $\frac{1}{2}$, this choice is not correct.

Please return to page $\frac{99}{2}$ and try question 20 again.

There are only two numbers with this special property:

$$1 \times 1 = 1$$

and

$$0 \times 0 = 0$$

No other number has this property.

Therefore, this choice is correct,

since

0 is less than 10 .

Please proceed to question 18 below.

Question 18

Apply your knowledge to find which statement is correct.

(A) $\frac{6 + 4}{2} = 3 + 4$

(B) $\frac{6 + 4}{2} = 3 + 2$

(C) $\frac{6 + 4}{2} = 6 + 2$

(D) None of these.

$\frac{94}{1}$

In adding fractions, it is necessary to use a common denominator. In this case, the common denominator would be 77 , and there is a good bit of arithmetic in completing the addition. However, it isn't necessary to do the addition to find the answer to this question. The fraction $\frac{22}{7}$ is larger than 3 , and adding a quantity to it gives a still larger result. Then it is not possible for the total to be equal to 2 .

Therefore, this choice is not correct. Incidentally, if you are interested, the total of the two fractions is

$$\frac{291}{77}$$

or

$$3\,\frac{60}{77}$$

Please return to page $\frac{83}{2}$ and try question 12 again.

$\frac{94}{2}$

One of the other choices is correct, therefore, this is not correct.

Please return to page $\frac{100}{2}$ and try question 15 again.

A simple rule which you should know is that if two fractions have the same numerator, the one with the larger denominator is the smaller fraction.

Therefore, $\frac{5}{13}$ is smaller that $\frac{5}{12}$ and this choice is correct.

Please proceed to question 10 below.

Question 10

The number

$$S = \frac{1}{2} + \frac{1}{3}$$

and the number

$$T = \frac{1}{2} \times \frac{1}{3}$$

Perform the calculations to find the correct choice.

(A) $T = S$

(B) $T > S$

(C) $S > T$

(D) None of these.

96/1

Since

$$6.2 \times 70 = 434$$

and

$$350 + 100 - 16 = 434$$

this choice does not make the inequality true.

Therefore, this choice is not correct.

Please return to page 104/2 and try question 14 again.

96/2

Finding the value of the numerator first, we get zero.

Then zero divided by 3 is 0 , which is not larger than 3 .

Therefore, this choice is not correct.

Please return to page 102/2 and try question 19 again.

In order for a division to give a result larger than 6, after dividing by 2; the number had to be larger than 12 . But the numerator could not be larger than 12 unless the question mark is replaced by a number larger than 9 . Since no such number was offered, this choice is the only correct one.

Please proceed to question 17 below.

Question 17

You know that

$$1 \times 1 = 1$$

Therefore, this is a number which when multiplied by itself gives the same number. Which of the following statements do you recognize is true?

(A) There is no other number like this.

(B) There are many other numbers like this.

(C) There is only one other number like this and it is larger than 10 .

(D) There is only one other number like this and it is less than 10 .

98
1

If you are sharp, you can save arithmetic on this one.

You know that 4 quarters equal one dollar. Then

$$25 \times 4 = 100$$

Since 25.47 is larger than 25 , when you multiply by 4 the result is larger than 100.

Therefore, this choice is correct, and it is not even necessary to perform the complete multiplication.

Please proceed to question 13 below.

98
2

Question 13

Which numeral do you recognize may replace the question mark so as to make the following statement true?

$$\frac{3 + 2}{2} = ? + 1$$

(A) 1.5

(B) 2.5

(C) 3

(D) None of these.

Finding the value in the numerator first, we get 11 .

Now

11 divided by 3 is larger than 1

therefore, this choice is correct. If you foolishly decide to divide before doing the subtraction, you must make sure to divide each number by the 3 . You would still find that this choice is correct, but subtracting first is a safer procedure.

Please proceed to question 20 below.

Question 20

Apply your knowledge to find the statement in which the question mark can correctly be replaced by the sign $<$.

(A) $\frac{1}{2} + \frac{1}{2} \; ? \; \frac{1}{2}$

(B) $\frac{3}{5} \times \frac{4}{3} \; ? \; 1$

(C) $\frac{5}{6} - \frac{1}{3} \; ? \; \frac{1}{2}$

(D) $1.6 \times .8 \; ? \; 1$

$\frac{100}{1}$

Since

$$6.2 \times 70 = 434$$

and

$$350 + 70 = 420$$

this choice makes the inequality true.

Therefore, this choice is correct.

Please proceed to question 15 below.

$\frac{100}{2}$

Question 15

Perform the calculation to find which number may replace the question mark to make the inequality true:

$$5 \times ? + 2 < 4$$

(A) 1

(B) $\frac{1}{2}$

(C) $\frac{1}{5}$

(D) None of these.

If you examine each element of this set, you will discover that every number except one possesses a certain property. Can you find this property?

Please return to page $\frac{110}{2}$ and try this question again.

What is the meaning of the three dots?

As you continue your study of mathematics, you will be impressed with the fact that mathematicians always look to express themselves more briefly. This is done by using symbols.

Thus, to enumerate the set of all natural numbers between 1 and 100 inclusive, we write

$$1 , 2 , 3 , . . . , 100$$

The three dots are a symbol which indicates all the numbers up to 100 .

Please return to page $\frac{127}{2}$ and try this question ag

The proper procedure is to find the value of the numerator first, then divide. If you do this, you will find that the division gives the result 5 , and the addition on the right side gives the result five.

Therefore, this choice is correct. Some students decide to divide before performing the addition. While foolish, it can be done if you are careful. You would have to divide <u>each</u> number in the numerator by the 2 , not just one of them. Adding first avoids any complication.

Please proceed to question 19 below.

<u>Question 19</u>

Apply your knowledge to find in which statement the question mark can correctly be replaced by the sign $>$.

(A) $\frac{15 - 4}{3}$? 1

(B) $\frac{6 + 4}{6}$? 5

(C) $\frac{2 \times 5 \times 0}{3}$? 3

(D) $\frac{1}{2} \div 2$? 2

We do not agree.

Four of the five elements of this set have something in common. One element describes something entirely different.

Please return to page $\frac{116}{2}$ and try this question again.

The set of whole numbers between 100 and 115 inclusive, consists of these two numbers and all the counting numbers between 100 and 115.

Ask yourself the following question:

Is $\frac{225}{2}$ a counting number?

Please return to page $\frac{112}{2}$ and try this question again.

I

Since

$$\frac{3 + 2}{2} = \frac{5}{2} = 2.5$$

and

$$1.5 + 1 = 2.5$$

it follows that this choice is correct.

Please proceed to question 14 below.

Question 14

$$6.2 \times 70 = 350 + ?$$

Which numeral do you recognize may replace the question mark to make the inequality true

(A) 84

(B) 70

(C) 12×7

(D) $100 - 16$

In multiplying decimals, the rule is to multiply the numbers ignoring decimals, and then add the number of decimal places in each of the numbers you were given in order to get the number of decimal places in the result.

Multiplying

16 by 8 we get 128

Since each number had one decimal place, the result has

1 + 1

2 decimal places

Therefore, the result is 1.28 , which is not smaller than 1.

Then this choice is not correct.

Please return to page 99/2 and try question 20 again.

Did you enumerate the elements of the set of natural numbers that are less than 25 ? If you didn't, we will do so now.

They are:

$$A = \{1 , 2 , 3 , \ldots , 24\}$$

Now, select the members of this set A that can be divided by 3 without a remainder.

Please return to page 129/1 and try this question again.

106
1

When we say that a set is well-defined, we mean that we can determine all of its elements. In order to do this, there should be no ambiguity about what element is or is not a member of the set. Now, the expression " large number " is subject to opinion. You may consider one million as a large number, but the Treasury Department may not. Hence, the set of large numbers is not well-defined.

Please return to page 122/2 and try this question again.

106
2

We don't agree.

A set which contains a large number of elements is not an infinite set. As long as the number of elements of a set can be specified the set is not infinite.

Please return to page 131/2 and try question 8 again.

$$5 \times \frac{1}{5} = 1$$

and

$$1 + 2 = 3$$

which is less than

$$4$$

Therefore, this choice is correct.

Please proceed to question 16 below.

Question 16

Perform the calculation to find which number may replace the question mark to make the inequality true:

$$\frac{? + 3}{2} > 6$$

(A) 7

(B) 8

(C) 9

(D) None of these.

The rule for dividing fractions is

" Invert the divisor and multiply ".

Then $\frac{1}{2}$ divided by 2 equals $\frac{1}{2}$ multiplied by $\frac{1}{2}$, and this has the value

$$\frac{1}{2} \times \frac{1}{2}$$

which is

$$\frac{1}{4}$$

It is not larger than 2 .

Therefore, this choice is not correct.

Please return to page $\frac{102}{2}$ and try question 19 again.

We do not agree.

A finite set is a set that contains a specific number of elements. The specified number may be very large as, for example, the set of all counting numbers from 1 to 1 billion. However, as long as the number of elements can be specified that set is finite.

Now can you specify the number of elements in set P ?

Please return to page $\frac{115}{2}$ and try question 7 again.

In multiplying two fractions, we should reduce when possible before doing the actual multiplication. Then the work would look this way:

$$\frac{1}{5} \times \frac{4}{1} = \frac{4}{5}$$

Since $\frac{4}{5}$ is less than 1 , this choice is correct.

You have now finished this Segment. Hand in the Punch Card.
You should have entered in your notebook the following:

When a calculation is performed there is always a single result if there is any. In subtraction, there is no result if the number being subtracted is larger than the number from which we are subtracting. In division, the divisor may not be zero.

You should now be able to complete the homework assignment
Number 1 , examples 6-10

Volume 1 Segment 4 begins on page 110.

110
1

Volume 1 Segment 4 begins here

Obtain a punch card from your instructor. In addition to the other identifying information that must be furnished by you, you are asked to punch out the following:

COLUMNS	48	and	50	0 4	(Sequence Number)
	54	and	56	0 4	(Type of Punch Card)
	60	and	62	0 1	(Volume Number)
	66	and	68	0 4	(Segment Number)

Your reading assignment for this segment is pg: 10 - 14

You have used the word Set on many occasions without being aware of the fact that this word has great significance in practically all branches of mathematics. You may have made the following statements:

I lost my pencil set.

I have a good set of teeth.

My mother has a beautiful set of dishes.

and so on. In this segment you will learn the meaning and use of the word set in mathematics.

You will now be asked a series of questions to draw your attention to the more important points.

110
2

Question 1

Apply the proper principle and select the element that does not belong to the set A :

$$\{2 \ , \ 4 \ , \ 6 \ , \ 7 \ , \ 8\}$$

where A is the set of even numbers < 10

(A) 8

(B) 7

(C) 4

(D) 2

111
1

Sorry, but we do not agree. The elements of set S show no discernable pattern. Since this set cannot be described, we cannot be sure what elements belong or do not belong to this set.

Please return to page 122
2 and try this question again.

111
2

In listing the elements of a set, we do not repeat the like elements. Thus, for example, if we were listing the set of flowers in a vase which had three roses, five violets, and two lilies; we would only list rose, violet, lily.

Please return to page 134
1 and try this question again.

I

112
1

Very good. You made the correct choice. The three dots after the number 8 mean that we continue to count all the even numbers until we reach 22 . In other words, the first few numbers give us the clue to the pattern; and the last number tells us how far to continue.

Thus, this set consists of

2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, and 11 is the correct answer.

Please go on to question 5 below.

112
2

Question 5

Apply the proper principle and state whether

I. $\frac{220}{2}$ II. $\frac{225}{2}$

are members of the set of whole numbers between 100 and 115 indecisive. Select the letter which labels the correct statement.

(A) I and II are members of this set.

(B) I and II are not members of this set.

(C) I is a member and II is not.

(D) I is not a member and II is a member.

Very good. You made the correct choice. The set of all natural numbers that are divisible by 5 is the set

$$\{5, 10, 15, 20, \ldots\}$$

Since there is no limit to the number of elements of this set, it is an infinite set. Similarly, the set of all prime numbers is

$$\{2, 3, 5, 7, 11, 13, \ldots\}$$

It is the set of all whole numbers that have no exact divisors except 1 and the number itself. Euclid, a mathamatician of ancient Greece, was the first man to prove that there is an infinite number of primes. Hence, this is an infinite set.

Please go on to question 9 below.

Question 9

Apply the proper principle and select the letter next to which all the statements about the following sets are correct.

$$P = \{5, 10, 15, 20, \ldots, 550\}$$
$$Q = \{2, 4, 8, \ldots, 128\}$$
$$R = \{3, 7, 11, 15, \ldots, 31\}$$

(A) $100 \in P$, $32 \notin Q$, $21 \in R$.

(B) $55 \in P$, $32 \in Q$, $24 \notin R$.

(C) $100 \notin P$, $64 \in Q$, $23 \in R$.

(D) $150 \in P$, $24 \in Q$, $27 \in R$.

114
1

The set $\{0\}$ contains one element. This element is a member of the set of whole numbers. As long as a set contains one or more elements, it is not the null set.

Please return to page 125/2 and try this question again.

114
2

What do we mean when we say that two sets are equivalent ? Consider, for example, the set of seats in your math class, and the set of students that attend this class. If every student in the class has a seat and all are present and there are no vacant seats left; then the set of seats and the set of students are equivalent sets.

Two sets are called equivalent if for every element of one set, we can match an element of the second set.

Please return to page 139/2 and try this question again.

Very good. You made the correct choice. Let us list all the elements of this set. They are,

$$A = \{1, 2, 3, \ldots, 24\}$$

The first element that is divisible by 3 is the element 3 . The next one is 6 . Yes, every time we add 3 to an element divisible by 3 , we get another element that is divisible by 3 .

Thus, $B = \{3, 6, 9, 12, 15, 18, 21, \text{and } 24\}$

The elements of this set are divisible by 3 . Hence, 8 is the correct answer.

Please go on to question 7 below.

Question 7

Apply the proper principle and select the finite set (or sets) from the following:

$P = \{$The set of all mixed numbers between 1 and 2$\}$

$Q = \{1, 2, 3, 4, \ldots, 10000\}$

$R = \{$The set of all whole numbers greater than 1 million.$\}$

Choose the letter which labels the correct statement.

(A) Set p is the only finite set.

(B) Set P and Q are finite.

(C) Set Q is the only finite set.

(D) All the sets are finite.

I

116
1

Very good. You made the correct choice. Every element of this set except the element 7 , is a natural number divisible by 2 . That is, dividing each number (except 7) by two, results in a natural number plus a zero remainder.

Please go on to question 2 below.

116
2

Question 2

Apply the proper principle of description of the set and select the one element that apparently does not belong to the set

{French, German, English, London, Italian}

(A) German

(B) Italian

(C) London

(D) English

I

We do not agree. Consider the number $\frac{220}{2}$. It is a natural number? Does it lie between 100 and 115 ? As soon as you find the answers to these questions, you will be able to select the correct letter.

Please return to page 112/2 and try this question again.

How many natural numbers less than 19 are divisible by 2 ? Did you write down all the elements of this set ? Please do so.

Return to page 134/1 and try this question again.

I

118
1

What is your definition of tall ? Some people may consider 6 feet as being tall, while other may feel that one should be 6 feet and 3 inches in order to be considered tall. Since the word tall is not well-defined, the set R is not a well-defined set.

Please return to page 122/2 and try this question again.

118
2

The symbol $\in$ means "belongs" or "is a member of."
Thus, we can say $3 \in S$, where

$$S = \{1, 2, 3, 4, 5,\}$$

This same symbol with a slash through it, $\notin$ means does not belong or it not a member of. Thus, $6 \notin S$. Now that you know the meaning of these symbols, you should be able to answer this question.

Please return to page 113/2 and try this question again.

119/1

We do not agree. To describe the elements of a set accurately, is to specify a characteristic that they have in common. You should look for a characteristic that distinguishes these months from all other months of the year.

Please return to page 136/2 and try this question again.

119/2

We don't agree. Keep this in mind.

A finite set cannot be equivalent to an infinite set.

One of the sets M, N, P is infinite.

Please return to page 133/2 and try this question again.

I

$\frac{120}{1}$

Sorry, but your choice is not correct. A finite set is a set that contains a specified number of elements. Consider the set of mixed numbers.

$$(1\tfrac{1}{2}) \ , \ (1\tfrac{1}{3}) \ , \ (1\tfrac{1}{4}) \ , \ (1\tfrac{1}{5}) \ , \ \ldots$$

Each one of these mixed numbers is greater than 1 and less than 2 . How many of these can you write?

Yes, you can go on and on. Hence, P is not a finite set.

Please return to page $\frac{115}{2}$ and try question 7 again.

$\frac{120}{2}$

The set of primes consists of all whole numbers that are divisible only by 1 and by themselves. Since 2 is divisible by 1 and by 2 , it is a prime number. Thus, the set Z is not the null set.

Please return to page $\frac{125}{2}$ and try this question again.

121
1

One ton is 2000 pounds. Have you ever seen a dog that weighs 2000 pounds?

Please reconsider you choice.

Please return to page 134 (1) and try this question again.

121
2

We disagree. Find the number of elements in each set.

If two of them have the same number of elements, then they are equivalent.

Please return to page 139 (2) and try question 14 again.

I

Very good. You made the correct choice. The elements French, German, English, and Italian can be thought of as languages. We can, for example, consider these elements as forming the set of languages taught in a high school. On the other hand, London is a city and, therefore, does not belong to this category.

Please go on to question 3 below.

Question 3

Apply the proper principle and select from the following the set which is well-defined:

P = {The set of large numbers.}

Q = {The set of presidents of the United States of America in office before 1968.}

R = {The set of tall students in your class.}

S = {7, 1, 16, 13}

(A) P

(B) Q

(C) R

(D) S

$\frac{123}{1}$

In order for $\frac{225}{2}$ to be a member of a set that consists of natural numbers, it must be a natural number; a counting number. Is it? On the other hand, when you reduce $\frac{220}{2}$ don't you get a natural number?

Please return to page $\frac{112}{2}$ and try this question again.

$\frac{123}{2}$

Think of the coins as being different. Let one of the coins be a penny and the other a nickel. Keep in mind that each coin can fall in two different ways.

Please return to page $\frac{128}{2}$ and try this question again.

I

124
1

We do not agree. The set P is composed of all natural numbers between 5 and 550 inclusive, that are multiples of 5 .

Is 100 a multiple of 5 ?

Yes, since $100 = 20 \times 5$

Please continue.

Please return to page 113/2 and try question 9 again.

124
2

It is true that the number of books in the main library is quite large, and so is the number of people living in the United States. However, unless these numbers are exactly alike, the sets are not equivalent.

Please return to page 133/2 and try this question again.

I

Very good. You made the correct choice. The set of distinct letters needed to spell Mississippi is

$$\{m, i, s, p,\}$$

Hence, this set has 4 elements.

The set of natural numbers less than 19 that are divisible by 2 is

$$\{2, 4, 6, 8, 10, 12, 14, 16, 18\}$$

Hence, this set has 9 elements.

The set of dogs that weigh 1 ton = 2000 pounds has no elements.

The word "republican" consists of 10 letters with no letter repeated. Hence, this set has the greatest number of elements. By the way, can you find another 10-letter word with all of its letters different? It is not easy. Another such word is phlegmatic.

Please go on to question 11 below.

Question 11

Apply the proper principle and select the null set (or sets) from the following sets:

$$X = \{0\}$$

$$Y = \{\text{The set of natural numbers less than 1 .}\}$$

$$Z = \{\text{The set of primes divisible by 2 .}\}$$

Choose the letter which labels the correct statement.

(A) X and Y

(B) X and Z

(C) Y

(D) Y and Z

I

126
1

How many numbers are there greater than 1 million ? No matter how large a number you write, it is always possible to write a larger number simply by adding one to it.

Please return to page 115/2 and try question 7 again.

126
2

The set of all stars in the sky is a very large set. However, if all the stars were counted, we would have a specified number. Thus, this set is finite. On the other hand, the set of all even numbers is infinite.

Please return to page 142/2 and try this question again.

Since the conditions for being included or excluded from the set are quite specific, this set is said to be "well-defined." Therefore, this is the correct answer.

Please go on to question 4 below.

Question 4

Apply the proper principle and find the number of elements of the set

$$T = \{2, 4, 6, 8, \ldots, 22\}$$

Select the letter which labels the correct statement.

(A) 22

(B) 11

(C) 7

(D) None of these.

I

128
1

Very good. You made the correct choice. These 4 months are different from the other 8 months of a year in the fact that they are the only ones having 30 days.

Please go to question 13 below.

128
2

Question 13

Apply the proper principle and select the letter which lists all the elements of the set described by the rule:

The set of all possible outcomes of tossing two coins

(A) {Heads, Tails}

(B) {2 heads, 2 tails}

(C) {2 heads, 2 tails, 1 tail and 1 head}

(D) {2 heads, 1 head and 1 tail, 1 tail and 1 head, 2 tails}

I

This is the correct answer.

Please go on to question 6 which follows.

Question 6

Apply the proper principle and find the number of elements in the set of natural numbers that are less than 25 and are divisible by 3 . Select the letter which labels the correct statement.

(A) 8

(B) 7

(C) 12

(D) 10

Sorry, but your choice is not correct. Examine the set R . How is this set generated ? Can you see a pattern ? Compare each number with the one following it.

Please return to page 113/2 and try this question again.

I

130/1

We do not agree. The number 2 is a prime.

Please return to page 125/2 and try this question again.

130/2

We do not agree. Please fill in all the members of this set until you reach 233 . Check your addition.

Please return to page 144/2 and try this question again.

I

Very good. You made the correct choice. Of the three given sets, Q is the only finite set. A finite set has a specified number of elements. The set Q consists of all the natural numbers between 1 and 10000 inclusive. This is a specified number.

Please go on to question 8 below.

Question 8

Apply the proper principle and select the infinite sets from the following:

X = {The set of all natural numbers divisible by 5 .}

Y = {The set of all whole numbers less than 1,000,000.}

Z = {The set of all prime numbers.}

Choose the letter which labels the correct statement.

(A) Y is an infinite set.

(B) X and Y are infinite sets.

(C) Y and Z are infinite sets.

(D) X and Z are infinite sets.

I

Very good. You made the correct choice. The sets X and Z are infinite sets. When are two infinite sets equivalent ? This is a very interesting question.

Infinite sets are said to be equivalent if the members of the two sets can be put in a 1- to -1 correspondence; that is, if we can match each element of one set with an element of the second set and vice versa.

Thus,

$$X = \{1, 2, 3, 4, \ldots\}$$
$$\updownarrow \quad \updownarrow \quad \updownarrow \quad \updownarrow$$
$$Z = \{2, 4, 6, 8, \ldots\}$$

Since we can always match an even number with a natural number, the sets are equivalent. The unusual feature is that we can say that there are as many even numbers as there are natural numbers.

Please go on to question 17 below.

Question 17

Apply the proper principle and find the cardinal number of the set that consists of the number of ways we can obtain a sum of 7 with one toss of two dice. Select the letter which labels the correct statement.

(A) 3

(B) 4

(C) 5

(D) 6

Very good. You made the correct choice. Two sets are called equivalent if we can match every element of 1 set with an element of the second set. When dealing with finite sets, it is easy to determine whether they are equivalent. All we have to do is to count the members of each set. If they have the same number, they are equivalent.

Please go on to question 15 below.

Question 15

Apply the proper principle and select the sets that are equivalent from the following sets:

M = {The set of all natural numbers divisible by 3 .}

P = {The set of all people living in the United States}

N = {The set of all books in the main library.}

Select the letter which labels the correct statement.

(A) All three sets are equivalent.

(B) N and P are equivalent

(C) None of the sets are equivalent.

(D) M and P are equivalent.

I

134
1

This is the correct answer.

Please go on to the next question.

Question 10

Apply the proper principle and select the set that has the greatest number of elements.

S = {The set of letters that spell Mississippi}

T = {The set of natural numbers less than 19 and divisible by 2 .}

U = {The set of dogs that weigh 1 ton.}

V = {The set of letters that spell Republican.}

Choose the letter that labels the correct statement.

(A) S

(B) T

(C) U

(D) V

134
2

Your answer is incorrect. Have you forgotten the meaning of "between"?

Please return to page 149/1 and try question 1 again.

Sorry, we don't agree. Did you try listing every combination of two digits ?

Please return to page 151/2 and try question 3 again.

Sorry, we disagree. In mathematics if one example can be found as an "exception to a proposed rule" then the rule is out. The number 4 is an even integer but it is not divisible by 30.

Please return to page 161/2 and try question 5 again.

I

136
1

Very good. You made the correct choice.

The set of natural numbers start with 1. Thus, there are no natural numbers less than 1.

Therefore, this set is empty. A set with no member is called the null set.

Please go on to question 12 below.

136
2

Question 12

Apply the proper principle and select the letter which most accurately describes the following set designated by the roster method:

$$T = \{\text{September}, \text{April}, \text{June}, \text{November}\}$$

(A) Four months of the year

(B) The months that have 30 days

(C) The months that have the most legal holidays

(D) None of these

Sorry, your answer is not correct.

The NULL SET, or the EMPTY SET, as it is sometimes called, is a subset of every set except itself.

Please return to page 158/2 and try question 2 again.

It's easy to be careless.

Have you forgotten that every set is a subset of itself?

Please return to page 166/2 and try question 6 again.

I

138
1

The set of natural numbers that are divisible by 5 are an infinite set. On the other hand, the population of the United States is a specified number and is not infinite.

Remember, a finite set cannot be equivalent to an infinite set.

Please return to page 133/2 and try this question again.

138
2

By the cardinal number of a set, we mean the number of elements that it has. Hence, the cardinal number of this set is the number of ways of obtaining a 7 with a single throw of 2 dice.

There are more than the number of ways you chose.

Please return to page 132/2 and try this question again.

Very good. You made the correct choice.

Let us suppose that one coin is made of silver and one is made of copper. Now, each coin has two faces and can therefore come up in 2 different ways. Thus, we can tabulate the possibilities as follows:

Silver	Copper
head	head
tail	tail
head	tail
tail	head

Please go on to question 14 below.

Question 14

Apply the proper principle and select the two sets that are equivalent from the following sets:

S = {The set of all natural numbers greater than 7.5 and less than 19.3}

T. = {1 , 4 , 9 , 16 , 25 , . . . , 100}

U = {The set of the months of a year.}

Choose the letter which labels the correct statement.

(A) S and T

(B) S and U

(C) T and U

(D) None of these.

Congratulations!

There are 10 subsets of set R which contain two elements each.

These subsets are:

$\{5, 6\}$ $\{6, 7\}$ $\{7, 8\}$ $\{8, 9\}$
$\{5, 7\}$ $\{6, 8\}$ $\{7, 9\}$
$\{5, 8\}$ $\{6, 9\}$
$\{5, 9\}$

Please continue below with question 4.

Question 4

S is the set $\{1, 3, 4, 8\}$

Perform the calculation to find the total number of subsets of S .

(A) 16

(B) 15

(C) 11

(D) 7

Have you forgotten?

Every set is a subset of itself.

Please return to page 158/2 and try question 2 again.

Have you forgotten the meaning of " proper subset "?

Please go back and look it up.

Return to page 155/2 and try question 9 again.

142
1

This is the correct answer.

Please go on to question 16 which follows:

Question 16

Apply the proper principle and choose the sets that are equivalent from the following sets:

X = {The set of all counting numbers.}

Y = {The set of all the stars in the sky.}

Z = {The set of all even numbers.}

Select the letter which labels the correct statement.

(A) X and Z

(B) Y and Z

(C) X and Y

(D) None of these.

142
2

We don't agree on how many days there are in July.

Please return to page 177/2 and try question 11 again.

I

We disagree.

All the whole numbers between 3 and 9 include a number which is not an element of set P.

Please return to page 149/1 and try question 1 again.

Sorry, we disagree.

In mathematics, if only " one exception to a proposed rule " can be found, then the rule is disproved.

The number 10 is a multiple of 5 , but 10 is not divisible by 30.

Please return to page 161/2 and try question 5 again.

144
1

Very good. You made the correct choice.

First of all, we must know the meaning of " cardinal number of a set ". By the cardinal number of a set we mean the number of members that compose the set. Thus, for example $\{0, 1, 2\}$ has a cardinal number of 3. Now each die is numbered from 1 to 6 and a sum of 7 can be obtained in the following ways:

1st die	2nd die
1	6
2	5
3	4
4	3
5	2
6	1

That is, 6 ways in all. Hence, the cardinal number of this set is 6.

Please go on to question 18 below.

144
2

Question 18

Apply the proper principle and choose the letter which correctly answers the following: Consider the set of numbers

$$F = \{1, 1, 2, 3, 5, 8, 13, \ldots\}$$

Where each number (except the first 2) is formed by adding the 2 previous numbers, for example,

$$5 = 3 + 2, \quad 8 = 5 + 3 \text{ etc.}$$

What is the cardinal number of this set if the last member is 233 ?

(A) 12

(B) 13

(C) 14

(D) The number 233 is not an element of this set.

I

It's easy to be careless.

Have you forgotten that the null, or empty set is a subset of every set?

Please return to page 166/2 and try question 6 again.

Have you forgotten the meaning of the term " improper subset "?

Please go back and look it up.

Return to page 163/2 and try question 8 again.

I

$\frac{146}{1}$

It's easy to be careless.

Take another look at set N. Is zero an element of set N ?

Please return to page $\frac{150}{1}$ and try question 7 again.

$\frac{146}{2}$

Sorry, we disagree.

The odd multiples of 3 are such numbers as

3 , 9 , 15 , 21 . . . .

These numbers are not integral multiples of 6 , and thus cannot be elements of set W.

Please return to page $\frac{170}{2}$ and try question 10 again.

We disagree.

The odd integers between 2 and 10 are the integers,

3 , 5 , 7 , and 9 .

Every set is a subset of itself.

Please return to page 158/2 and try question 2 again.

Did you forget the empty set?

The empty set is a subset of all sets except itself.

Did you forget the set S itself?

Each set is a subset of itself except the empty set.

Please return to page 140/2 and try question 4 again.

Very good. You made the correct choice.

Did you fill in the members of the set until you reached 233 ? Let us do so together:

1 , 1 , 2 , 3 , 5 , 8 , 13 , 21 , 34 , 55 , 89 , 144 , 233

Hence, 13 is the cardinal number of this set. It is interesting to note that the set of numbers formed by the above rule is quite famous. The elements of this set are called Fibonacci Numbers. Those numbers have many interesting properties and in your future study of mathematics, you are going to hear more about these numbers.

You have now finished this segment. Hand in your punch card.

You should have entered in your notebook the following definitions and formulas:

(1) Definition of a set.

(2) Definition of a finite and infinite set.

(3) Definition of a null set.

(4) Definition of the cardinal number of a set.

(5) Definition of equivalent sets.

(6) Definition of Roster Method and rule for describing a set.

You should now be able to complete the following problems from your homework assignment: Problems 13 , 14 , 15 , and 16.

Volume 1 Segment 5 begins here:

Obtain a PUNCH CARD from your instructor. In addition to the other identifying information that must be furnished by you, you are asked to punch out the following:

COLUMNS	48	and	50	0 5	(Sequence Number)
	54	and	56	0 4	(Type of Punch Card)
	60	and	62	0 1	(Volume Number)
	66	and	68	0 5	(Segment Number)

Your READING ASSIGNMENT for this segment is pg: 18
You will now be asked a series of questions to draw your attention to the more important points.

Question 1

Which of the choices do you recognize as a subset of the set

$$P = \{4 , 6 , 7 , 8\} \quad ?$$

(A) $\{$all whole numbers between 4 and 8$\}$

(B) $\{$all whole numbers between 3 and 9$\}$

(C) $\{2 , 4 , 6 , 8\}$

(D) $\{4 , 6 , 7 , 8\}$

150
1

Your answer is correct.

Please go on to question 7 which follows:

Question 7

Which choice do you recognize as a subset of

$$N = \{1 , 2 , 4\} \quad ?$$

(A) $\{0 , 1\}$

(B) $\{0 , 1\}$.

(C) $\{0 , 1 , 2 , 4\}$

(D) None of these.

150
2

It's very easy to be careless.

You have evidently confused the terms " proper " and " improper " subsets.

Please return to page 155/2 and try question 9 again.

Congratulations! Your answer is correct.

Six is a one-digit multiple of three, and is therefore an element of the set described by choice B. Six is not an element of set Q. Thus, the set described in B cannot be a subset of Q.

Please go on to question 3 below.

Question 3

R is the set

$$\{5 , 6 , 7 , 8 , 9\}$$

Perform the calculation to find the number of subsets of R which contain 2 elements each.

(A) 10

(B) 8

(C) 4

(D) 2

152
1

In the Northern Hemisphere, summer arrives in late June and leaves in late September. In the Southern Hemisphere, summer arrives in late December, and leaves in late March.

Do you still think that November is a summer month?

Please return to page 177/2 and try question 11 again.

152
2

It's easy to be careless.

Please re-read choice A again.

Does 1st year algebra contain all the other subjects given in high school?

Please return to page 180/1 and try question 15 again.

I

You recognized that the null set is a subset of every set. However, your choice

B , $\{\emptyset , 1\}$ shows the null set as an

element of set N.

The null set is not an element of set N.

Please return to page 150/1 and try question 7 again.

Sorry, we disagree.

Some of the spectators in the PDQ seats might be the parents of the football players. These parents are not students at the college.

Please return to page 179/2 and try question 13 again.

$\frac{154}{1}$

It's very easy to be careless.

Is the element 2 in set P ?

Please return to page $\frac{149}{1}$ and try question 1 again.

$\frac{154}{2}$

We don't agree.

Try to list sets following a systematic pattern.

Please return to page $\frac{140}{2}$ and try question 4 again.

One of the subsets that may be formed from any set is that set itself, we call it an " improper subset ". Each set can have one, and only one, improper subset.

Your answer is correct.

Please go on to question 9 below.

Question 9

Perform the calculation to find the number of proper subsets of

$$V = \{a , e , i , o , u\}$$

(A) 0

(B) 1

(C) 32

(D) 31

156/1

Sorry, we disagree.

In mathematics, if only " one exception to a proposed rule " then the rule is out.

The number 40 is a multiple of 20 , but 40 is not divisible by 30.

Please return to page 161/2 and try question 5 again.

156/2

We fooled you this time.

The empty set contains no elements, but it is a subset of every set except itself.

The symbol for the empty set is $\emptyset$. The set $\{\emptyset\}$ is the set containing the empty set.

Please return to page 166/2 and try question 6 again.

Sorry, we disagree.

The odd multiples of 2 include such numbers as

2 x 5

2 x 7

2 x 11

2 x 13

These numbers:

10 , 14 , 22 , and 26

are not multiples of

6

and are not members of set W

Please return to page 170/2 and try question 10 again.

Sorry, we disagree.

Some holidays fall on Saturdays, others on Sundays.

Please return to page 183/2 and try question 14 again.

158
1

Congratulations! Your answer is correct.

Every set is a subset of itself.

Please go on to question 2 below.

158
2

Question 2

Which choice do you recognize is NOT a subset of set

$$Q = \{3 , 5 , 7 , 9\} \quad ?$$

(A) $\emptyset$

(B) {all multiples of 3 which contain one digit}

(C) $\{3 , 5 , 7 , 9\}$

(D) {all odd integers between 2 and 10}

Did you misread the question?

You were asked which set was set Z a subset of.

Z is not a subset of the empty set.

Please return to page 172/1 and try question 12 again.

Your choice was not correct.

The product of two odd numbers is an odd number not an even number.

Please return to page 168/2 and try question 16 again.

I

160/1

Haven't you forgotten something?

You found the number of total subsets possible. This includes improper as well as proper subsets.

What did the question ask you to find?

Please return to page 155/2 and try question 9 again.

160/2

Sorry, we disagree.

The set of two digit numbers contains many elements that are not the sum of two one-digit numbers. The largest sum of two one-digit numbers is

$$9 + 9 = 18$$

There are 81 other two digit numbers which don't fit the description.

Please return to page 181/2 and try question 17 again.

Congratulations! Your answer is the correct one.

To find the total number of subsets of a set you may wish to follow this procedure: Let n represent the number of elements in a set.

If n = 1 that is, for set T = $\{x\}$

the possible subsets are <u>two</u>: A = $\{x\}$ and B $\emptyset$

If n = 2 that is, for a set V = $\{x , y\}$

the possible subsets are <u>four</u>: A = $\{x\}$

B = $\{y\}$

C = $\{x , y\}$ and

D $\emptyset$

If n = 3 that is, for a set W = $\{x , y , z\}$

the possible subsets are <u>eight</u>: A = $\{x\}$ E = $\{x , z\}$

B = $\{y\}$ F = $\{y , z\}$

C = $\{z\}$ G = $\{x , y , z\}$ and

D = $\{x , y\}$ H = $\emptyset$

There is a definite pattern here,

<u>number of elements</u>	<u>use 2 in multiplication</u>		<u>number of subsets</u>
1	1	(2) =	2
2	2	(2)(2) =	4
3	3	(2)(2)(2) =	8
4	4	(2)(2)(2)(2) =	16
.....	.....		.....
n	n	2^n =	2^n

Please go on to question 5 below.

<u>Question 5</u>

T is the set

$\{$all the integers divisible by 30$\}$

Which of the following do you recognize is a subset of T ?

(A) $\{$even integers$\}$

(B) $\{$multiples of five$\}$

(C) $\{$multiples of 20$\}$

(D) $\{$multiples of 60$\}$

162/1

Sorry, we disagree.

One of the previous three choices is the correct one.

Please return to page 177/2 and try question 11 again.

162/2

Sorry, we disagree.

The S is not a subset of the set containing the empty set.

Please return to page 180/1 and try question 15 again.

Your answer is correct.

None of the choices, A , B , or C is a subset of set N.

Choices A and C , have zero as elements of the subsets, yet zero is not an element of N.

Choice B has the empty set as an element, and the empty set is not an element of N.

Congratulations!

Now please go on to question 8 below.

Question 8

Perform the calculation to find the number of improper subsets of

$$P = \{2.4 , 2.5 , 2.7 , 2.8\}$$

(A) 0

(B) 1

(C) 10

(D) 16

$\frac{164}{1}$

When we say divisible by 2 , we mean that a number may be divided by two without leaving a remainder.

The number 1 does not fit that description.

Please return to page $\frac{172}{1}$ and try question 12 again.

$\frac{164}{2}$

Sorry, we disagree.

The sum of two odd numbers is an even number.

Please return to page $\frac{168}{2}$ and try question 16 again.

Whose band played at half-time?

Sometimes a local high school band is invited to play during half-time. The members of the high school band would not be students at the college.

Return to page 179/2 and try question 13 again.

Sorry, we don't agree.

What is the square of 6 ?

Please return to page 173/2 and try question 18 again.

I

166
1

Congratulations! Your answer is correct.

Perhaps you would like to see one way in which this problem can be handled.

The set T can be written in roster form as follows:

$T = 30 , 60 , 90 , 120 , 150 , 180 , 210 , 240 , 270 , 300 ,$
$330 , 360 , 390 , 420 , 450 , . . .$

The set of multiples of 60 can be written:

$X = 60 , 120 , 180 , 240 , 300 , 360 , 420 , . . .$

Beginning with the second element of T , every other element of T belongs to X.

Thus, the set of multiples of 60 is a subset of T.

Please go on to question 6 below.

166
2

Question 6

Which choice do you recognize as containing all the subsets of

$M = \{3 , 5\}$?

(A) $\{3\} , \{5\}$

(B) $\{3\} , \{5\} , \{3 , 5\}$

(C) $\{3\} , \{5\} , \{3 , 5\} , \emptyset$

(D) $\{3\} , \{5\} , \{3 , 5\} , \{\emptyset\}$

I

Sorry, we don't agree.

One of the choices is a subset of W.

Please return to page 170/2 and try question 10 again.

We disagree here.

Some weekdays are not school days. Election Day is a weekday yet there is no school on Election Day.

Please return to page 183/2 and try question 14 again.

I

Congratulations!

First year algebra is one of the subjects given in high school; thus, the set containing first year algebra is a subset of the set of all subjects given in high school.

Please go on to question 16 below.

Question 16

Which of the following statements do you choose as correct?

(A) $\{5, 11, 17\}$ is a subset of $\{2, 5, 8, 11, \ldots\}$

(B) $\{\text{product of two odd numbers}\}$ is a subset of $\{\text{even numbers}\}$

(C) $\{\text{sum of two odd numbers}\}$ is a subset of $\{\text{odd numbers}\}$

(D) $\{0\}$ is a subset of $\{\text{whole numbers between 5 and 6}\}$

It's easy to be careless.

How many digits are there in the number 10 ?

Please return to page 172/1 and try question 12 again.

We disagree.

If a number is an element of a subset of a set, then it must be an element of the larger set, too.

The number 1 will always divide another number. Thus, 1 will be an element of S and thus of T.

Please return to page 178/2 and try question 19 again.

I

170
1

The number of proper subsets is one less than the total number of subsets because the set itself is counted as a subset, but it is an improper subset. When a set has five elements, the number of total subsets is found by multiplying five 2's:

$$2 \times 2 \times 2 \times 2 \times 2 = 32$$

Now, deduct 1 from that figure because one of those subsets will not be a proper subset.

Thus,

$32 - 1 = 31$ represents the number of proper subsets of set V.

Your answer was correct. Congratulations!

Now please continue with question 10 below.

170
2

Question 10

If

$$W = \{\text{multiples of } 6\}$$

apply your knowledge to find which choice is a subset of W.

(A) $\{\text{even multiples of } 3\}$

(B) $\{\text{odd multiples of } 3\}$

(C) $\{\text{odd multiples of } 2\}$

(D) none of these

Sorry, but we disagree. There are many two digit numbers which are not the sum of two one-digit numbers.

Please return to page 181/2 and try question 17 again.

I'm sorry, we disagree. What integer squared, gives 3 as a result ?

Please return to page 173/2 and try question 18 again.

I

172
1

Your answer is correct.

Please go on to question 12 which follows.

Question 12

Set $Z = \{1, 2, 10\}$

is a subset of one of the following sets.

Which do you recognize as correct?

(A) $\emptyset$

(B) $\{$one digit numbers$\}$

(C) $\{$numbers divisible by 2$\}$

(D) $\{$numbers with 3- letter English names.$\}$

172
2

Sorry, we disagree. The empty set is a subset of every set. However, the set containing the empty set is NOT a subset of every set.

To designate the empty set, we use the symbol, $\emptyset$.

To designate the set containing the empty set, we use the notation $\{\emptyset\}$.

Please return to page 180/1 and try question 15 again.

I

Congratulations! The elements of set T contain the numbers between 0 and 18, inclusive. Certainly, the numbers between 2 and 18 are in this interval. You realize that the subset doesn't have to include all of the possible numbers.

Please go on to question 18 below.

Question 18

The set

$$N = \{\text{squares of integers}\}$$

is a subset of R. Apply your knowledge to find which statement is correct.

(A) $R = \{\text{numbers whose last digit is } 0, 1, 4, \text{ or } 9\}$

(B) $R = \{\text{numbers whose last digit is } 0, 1, 4, 5, 6, 9\}$

(C) $R = \{\text{numbers whose last digit is } 0, 1, 2, 3, 5, 6, 9\}$

(D) $R = \{\text{numbers with any digit as last digit}\}$

174/1

Sorry, we disagree. You are confusing the set containing the element 0 with the empty set, $\emptyset$.

Please return to page 168/2 and try question 16 again.

174/2

It's easy to be careless.

The square root of

$$25 \text{ is } 5 ,$$

this cannot be an element of the set described in B. Thus, the square root of 25 would not be an element of N, yet this contradicts the statement of the problem.

Please return to page 185/2 and try question 20 again.

I

It's very easy to be careless. You found the total number of possible subsets, not the number of improper ones.

Please return to page 163/2 and try question 8 again.

It's easy to be careless. Are Memorial Day and New Year's Day the only holidays in the year ?

Please return to page 183/2 and try question 14 again.

I

176/1

Sorry, we disagree. One of the previous choices is the correct one.

Please return to page 153/2 and try question 13 again.

176/2

Sorry, we disagree. The number 19 is an element of the set described in C , yet, 19 does not belong to set T .

Please return to page 181/2 and try question 17 again.

I

Congratulations! Your choice is correct. One of the ways you might have determined this would be to realize that the set of even multiples of 3 , is formed by listing the elements:

3×2	(3×2)
$3 \times 2 \times 2$	$(3 \times 2) \times 2$
$3 \times 2 \times 2 \times 2$	$(3 \times 2) \times 2 \times 2$
$3 \times 2 \times 2 \times 2 \times 2$	$(3 \times 2) \times 2 \times 2 \times 2$

etc.

These elements are

$$6, 12, 18, 24, \ldots$$

or

$$6, 6 \times 2, 6 \times 3, 6 \times 4, \ldots$$

which are the elements of set W .

Please go on to question 11 below.

Question 11

Set Y = {July, November}

is a proper subset of one of the following sets.

Which do you recognize is correct?

(A) {30 day months}

(B) {summer months}

(C) {months containing more than 29 days}

(D) none of these

I

Congratulations! In the process of squaring any number, the first step will always be to get the product of the units digit. This will involve one of the following products:

$1 \times 1 = 1$	$6 \times 6 = 36$
$2 \times 2 = 4$	$7 \times 7 = 49$
$3 \times 3 = 9$	$8 \times 8 = 64$
$4 \times 4 = 16$	$9 \times 9 = 81$
$5 \times 5 = 25$	$10 \times 10 = 100$

The units digits of these squares are:

1, 4, 5, 6, 9, 0

Please go on to question 19 below.

Question 19

The last two digits of a number are 2 0 and its divisors are in set S . If S is a subset of T , apply your knowledge to find T .

(A) $\{2, 5, 10, 20\}$

(B) $\{1, 2, 5, 10, 20\}$

(C) $\{1, 2, 4, 5, 10, 20\}$

(D) None of these.

Congratulations! Your answer is the correct one. In English the names of the numbers,

1 is one

2 is two

10 is ten

There are three letters in each name.

Please go on with question 13 below.

Question 13

Which choice do you recognize as a subset of

{all the students at PDQ College} ?

(A) {The spectators in the PDQ seats at the football game.}

(B) {The players on the PDQ team at the football game.}

(C) {The players in the band at half time at the game.}

(D) None of these.

I

180
1

Your answer is correct.

Please go on to question 15 which follows.

Question 15

Set $S = \{\text{subjects given in high school.}\}$

Which choice do you recognize is a correct statement ?

(A) $\{S\}$ is a subset of $\{\text{1st Year Algebra}\}$

(B) $\{\text{1st Year Algebra}\}$ is a subset of $\{S\}$

(C) $\{S\}$ is a subset of $\{\emptyset\}$

(D) $\{\emptyset\}$ is a subset of $\{S\}$

180
2

We disagree. The number 4 , will always divide a number ending in 2 0 because,

20 divided by 4 is 5

120 divided by 4 is 30

220 divided by 4 is 55

320 divided by 4 is 80

and; if you are dividing by 4 they are the only possible combinations.

Please return to page 178/2 and try question 19 again.

I

Congratulations! The set

$$\{2, 5, 8, 11, \ldots\}$$

is an infinite set. We recognize a pattern, or rule for the formation of successive elements.

Each new element is three more than the previous. The next two elements in the set are 14 and then 17 . Thus, the set

$$\{5, 11, 17\}$$

is a subset of the set

$$\{2, 5, 8, 11, \ldots\}$$

Please go to question 17 below.

Question 17

The set

$$T = \{\text{sum of two one-digit numbers}\}$$

Apply your knowledge to find which choice lists the set which is a subset of T .

(A) $\{\text{two-digit numbers}\}$

(B) $\{\text{one or two-digit numbers}\}$

(C) $\{\text{numbers between 1 and 20}\}$

(D) $\{\text{numbers between 2 and 18}\}$

I

182
1

I'm sorry but we disagree. What integer squared gives 2 as a result?

Please return to page 173 2 and try question 18 again.

182
2

I'm afraid we have confused you. The empty set (referred to as the null set, and noted as ∅) is a subset of every other set, but it has no subsets of itself.

Please return to page 185 2 and try question 20 again.

I

Your answer is correct. According to the rules of the game.

In order to play on a college football team, you must be a student at that college.

Please go on to question 14 below.

Question 14

Which choice do you recognize as correct ?

(A) {holidays} is a subset of {weekdays}

(B) {weekdays} is a subset of {school days}

(C) {Memorial Day, New Year's Day} is a subset of {holidays}

(D) {holidays} is a subset of {Memorial Day, New Year's Day}

I

184
1

Sorry, the correct answer is one of the other three choices.

Please return to page 178 (2) and try question 19 again.

184
2

We disagree. One of the other choices is the correct one.

Please return to page 185 (2) and try question 20 again.

I

Congratulations! Your answer is the correct one. If the divisors of the number are in set S , then set T , having set S as a subset, must contain all possible divisors of 20 , too.

Please go on to question 20 below.

Question 20

The square of an integer N ends in 25 . Then N is a subset of one of the following.

Apply your knowledge to find the correct choice.

(A) $\{$numbers ending in 5$\}$

(B) $\{$numbers ending in 10$\}$

(C) $\emptyset$

(D) None of these.

Congratulations! Your answer is correct.

You have now finished Segment 5. Hand in the PUNCH CARD.

You should have entered in your NOTEBOOK the following definitions:

(1) The definitions of a subset.

(2) The definition of the empty set, or null set.

(3) The definition of proper and improper subset.

You should be able to complete the following problems from your HOMEWORK ASSIGNMENT; page 19 , questions 1 - 14

You are now advised to review all of your notes and to re-read the reading assignment in order to prepare yourself for the volume test.

PROGRAMMED MATHEMATICS CONTINUUM
ALGEBRA - LEVEL ONE

ERRATA SHEET
VOLUME 1

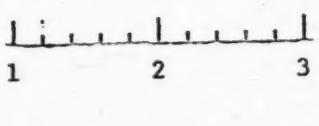
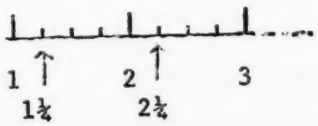
Attach to Back Cover

To the users of this book:

Computer analysis of the student's performance in his progress through this book will have as one of its purposes the collection of data indicating the need for revision of the material presented.

Certain typographical errors already exist and will also be corrected.

Listed below are misprints that will affect the mathematics of the problems. Make a careful correction of each misprint as follows:

PAGE	MISPRINT	CORRECTION	CHECK WHEN CORRECTION MADE
$\frac{34}{2}$	page $\frac{40}{9}$	page $\frac{40}{2}$	
$\frac{35}{1}$	see diagram	The line $4\frac{1}{2}$ should run from 7 to $2\frac{1}{2}$	
$\frac{39}{2}$	1 , 4	1.4	
$\frac{44}{2}$			
$\frac{46}{1}$	coefficients	coordinates	
$\frac{90}{2}$	$2 \div 2$	$2 \div .2$	
$\frac{91}{1}$	1	$1\frac{2}{3}$	
$\frac{104}{2}$	=	$\neq$	
$\frac{109}{1}$	$\frac{1}{5} \times \frac{4}{1}$	$\frac{1}{5} \times \frac{4}{1}$	
$\frac{112}{2}$	indecisive	inclusive	
$\frac{161}{1}$	B $\emptyset$ D $\emptyset$	B = $\emptyset$ D = $\emptyset$	
$\frac{69}{1}$ Ques. I-D:	~~1111 1111 1111~~	~~1111 1111~~ 1111	